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PERMANENT ADDRESS:

5 Darook St
Sth. Blackburn 3130
Victoria, Australia

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PATTERN PROCESSING AND ELEMENTARY SCHOOL MATHEMATICS

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Pattern Processing and Elementary School Mathematics" submitted by Bettina Mary Blackall in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

L. D. Nelson

Supervisor

Daiyo Sawada

Thomas E. Kieren

Lillian Whyte

Robert E. Reys

External Examiner

Date November 15th, 1974

ABSTRACT

Pattern involves classes, order and relations, three structures generally conceded to be the bases of mathematics. As well, they are embodied in the Piagetian concepts of conservation and classification, two mental operations postulated by the Genevans as necessary for mathematical understanding.

The present study was designed primarily to investigate the pattern processing abilities of elementary school children. Five pattern processes (interpolate, continue, reverse, select and generate), two forms (geometric and numerical) and two arrangements (linear and matrix) were incorporated into a hierarchy of 20 tasks. A secondary aim of the study was to determine the relationship, if any, between the children's pattern processing ability and their level of development in Piagetian conservation and classification at the stage when they are entering the period of concrete-operational thought.

A sample of 100 Grade three children was selected randomly from 11 classrooms within the Edmonton Public School System. No adaptation classes of specially chosen

pupils were included; however, no restriction was placed on children of low intelligence or physical handicap. The battery of tests administered to the sample included the Pattern Processing Test, Piagetian conservation and classificaiton tasks, and standardized tests in mathematics achievement and intelligence.

It was found that the Grade three pupils in the sample were able to solve a wide variety of pattern processing problems with varying degrees of success depending on the difficulty component built into the items. The performance of the children on the four generate tasks ranged from randomness to the production of sequences and matrices having more complex pattern descriptions than those presented by the investigator in the tasks with preset correct answers. Although a set of characteristics to distinguish the successful pattern processor does not emerge from this study, high performance on the Numerical: Matrix: Interpolate task, as well as high scores on mathematics achievement and intelligence, seem significant. Furthermore, it appears that a child at Grade three level is unlikely to be highly successful in pattern processing unless he has the ability to conserve area along with the classificatory skill of class inclusion.

Categorization of the error responses into a limited number of strategies indicated that the children tended to exhibit rigidity as well as logic of a sort in their unsuccessful efforts to solve pattern problems.

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Since the beginning of man in recorded history, the phenomenon of pattern has occupied an important place in his culture. Klahr and Wallace (1970) pointed out that "the ability to detect environmental regularities is a cognitive skill essential for survival (p.243)." They add, "Man has a propensity to seek and capacity to find serial patterns in such diverse areas as music, economics and the weather (p.243)." Nowhere is pattern more clearly displayed than in mathematics, whose relationships embodying order, rhythm and harmony can be seen in artistic representation of all ages from the cave paintings of the Australian aborigines, through the artists of the Renaissance, to modern sculptors like Rodin, Moore and Epstein.

According to Niman and Feldstein (1973), mathematics is itself an art, which "expresses beauty through a system of definitions, axioms and theorems (p.531)." Johnston (1968) added a warmth to his appreciation of mathematics as an art.

> The ideas, the logic, the patterns of mathematics fit together in an elegant, harmonious way like the sounds of a musical composition, the words of a poet, or the colours of a painting (p.328).

In his autobiography, Hardy (1967) claimed that "a mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas (p.84)."

That pattern is basic to mathematical relationships is confirmed time and time again by the great mathematicians. In fact, the philosopher-mathematician Alfred North Whitehead (1968) declared without qualification that "Mathematics is the study of pattern (p.106)." Moreover, he specified that "for the purposes of education mathematics consists of the relations of number, the relations of quantity, and the relations of space (1929, p.83)."

There seems to be no justification for Whitehead's view of mathematics being restricted to higher education or even to the secondary school. Yet such a notion of mathematics-in-pattern is not reflected in the widely used elementary programs, which encourage a minimal awareness of pattern. If pattern activities are included, they are usually found in the enrichment exercises for the most successful pupils. The young school child, not yet at the stage of abstract thought and not necessarily a high achiever, can hardly be considered a knowledgeable mathematician. Nevertheless, if mathematics is accepted as an essential part of the elementary school curriculum, he deserves the opportunity to experience mathematics through the study of pattern, an integral part of the mathematician's craft.

Further evidence to justify the claim that the topic of pattern has a legitimate place in the content of school mathematics is provided by the psychologists who concentrate on the epistemological aspects of mathematics. One such researcher is Scandura for whom the aim of mathematics education is not to produce mathematicians so much as to develop students "who think creatively, who can think like a mathematician (Scandura, 1971, p.6)." From his analysis of the mathematical process, he has listed six processing skills. The first of these is "the ability to detect regularities," which involves "perceiving a pattern or drawing out or abstracting that which is common to a number of examples(p.6)." Another psychologist, Margaret Donaldson (1963), used problems involving series extrapolation in order to examine children's thinking processes. She commented that successful solution is often based on "an inarticulate feeling for rhythm (p.137)."

Although it may be argued persuasively that experiences in pattern should be included in the content of elementary mathematics programs, their appropriateness to the child's level of cognitive growth should be considered as well. The psychological theory that seems to have most relevance to elementary school children is Jean Piaget's notion of developmental stages with specific reference to the advent of concrete-operational thinking, a necessary condition before a child can form abstract mathematical concepts (Piaget, 1953). Two characteristics of a child who has reached the concrete-operational stage are conservation and classification, mental structures which Piaget maintains are important for mathematical understanding. Piaget's claim is supported by other researchers such as Almy (1966), Cathcart (1969), Reimer (1968) and Riggs (1970).

In the introduction to The Early Growth of Logic in the Child (Inhelder & Piaget, 1969), a research report on the development of classification and seriation, it is stated that "a part of Piaget's thesis is that a correct understanding of classificatory relations is essential to an adequate conception of number (p.xvii)." With regard to conservation, the acknowledgement that a specific property of an object remains invariant despite transformation in another property, Piaget (1965) has written:

> In a word, whether it be a matter of continuous or discontinuous qualities, of quantitative relations perceived in the sensible universe, or of sets and numbers conceived by thought, whether it be a matter of the child's earliest contacts with number or of the most refined axiomatizations of any intuitive system, in each and every case the conservation of something is postulated as a

> necessary condition for any mathematical understanding (p.3).

Mathematicians of note have pointed out the intimate relationship between pattern and mathematics. Research, both in Geneva and elsewhere, has supported Piaget's contention that a child's mathematical understanding depends on his prior attainment of the mental structures of conservation and classification. It seems a natural extension of the relationships among pattern, mathematics and developmental psychology to consider the nature of the process of pattern and its place in a child's cognitive development at the stage where he is entering the period of concrete-operational thought. Thus we would be taking another step forward in our knowledge of how children learn mathematics.

THEORETICAL FRAMEWORK

Illustrations to support the contention that pattern is integral to human experience are followed by a summary of the generally inadequate psychological theories and research pertaining to pattern as a process. The major discussion will focus on the pattern process as a synthesis of the Mother Structures of the Bourbaki School of Mathematicians and Jean Piaget's mental structures within his theory of cognitive development. To conclude this section the hierarchy of tasks will be discussed, with particular reference to the contribution of information processing research.

Pattern and Human Experience

For the human neophyte, pattern has its beginnings in experience of the environment. The recurrent regularities of eating and sleeping, of daylight and darkness, extend to contact with the organic world where pattern can be seen in the stumps of trees, in the honeycomb, in the arrangement of the petals of a chrystanthemum. D'Arcy Thompson's On Growth and Form (1966) is devoted entirely to examples of environmental patterns which he often attempted to represent in geometric form. The biologically-oriented Piaget has commented that "every living being is arranged according to different plans of symmetry (Beth & Piaget, 1966, p.203)."

Complementary to the structures inherent in natural objects is the ability to recognize and distinguish different patterns, a fundamental property of living organisms, according to Dodwell (1970). Thus pattern can be considered a two-way contact between man and the natural world. Newell and Simon's (1972) statement that "man is the mirror of the universe in which he lives and all he knows shapes his psychology (p.866)" endorses the integral relation between pattern in the environment and human experience. Whitehead (1929) commented that "the notion of the importance of pattern is as old as civilization (p.109)." A natural parallel can be drawn between the generic characteristic of man to seek structure in the world (Wertheimer, 1970) and a child's increasing experience with pattern.

Pattern in the Psychology Schools

Since the rise of psychology as a discipline, the study of pattern has proceeded along two divergent paths. One group, especially the Gestaltists, concentrated on shape pattern within the field of perception, while the psychologists interested in measurement used patterns, usually in the form of numerical series, as the vehicle for intelligence testing. More recently, this kind of use has been extended to the study of thinking.

Perception and pattern. The Gestaltist School of Psychology, founded in Germany, maintained that there is an essential unity flowing through nature (Perkins, 1969), and that each natural phenomenon is "a whole, whose structure can be explained neither by the qualities of its single elements, nor by the relationships between these elements

(Arnheim, 1943, p.72)." The whole cannot be inferred from the parts taken separately. Thus the emphasis is on "organised wholes, separated from the other wholes but united within themselves by their dynamic patterning (Hill, 1971, p.96)." Because of their emphasis on wholes, the Gestaltists were not concerned with the pattern process. On the other hand, their "perceptual good form" causes no conflict with the idea of pattern as recurrent regularity.

A similar situation is observed in the work on perception by psychologists such as Gibson (1969), the Picks (1970), Vernon (1962, 1970) and Zaporophets (1970). They have looked at perceptual development and learning in which perception is the process. Where pattern appears in these studies, it is as a product or a vehicle for the overt manifestation of perceptual activity.

Furthermore, on the basis of extensive research, Piaget (1969) concluded that perceptual activity is not a sufficient condition even for the recognition of relations.

Pattern and the measurement of intelligence.

Since the turn of the century, pattern in both geometrical and numerical forms has been the means of much psychological measurement of intelligence. Tests with sections on pattern include the Weschler Intelligence Scale for Children, SRA's

Short Test of Educational Ability, the Kulmann-Anderson Test, the California Short-form Test of Mental Maturity, the Laycock Mental Ability Test, the Terman Group Test of Mental Ability, and Cattell's Culture Fair Test of g. Thus it is clear that psychologists consider pattern to be one of the keys to the measurement of cognitive functioning. Yet the authors of these tests seemed to assume that the solution of numerical relationships provides a measure of intelligence because such items differentiated between subjects. Goodness-of-fit of scores to a normal curve was apparently the criterion for item selection. None of the manuals for the tests mentioned include a discussion on why, for instance, number series are used. Writing in 1926, Thorndike conceded that "all scientific measurement of intelligence that we have at present are measures of some product produced by the person or animal in question (Thorndike et al., 1926, p.11)." However, Raven (1954) gave some rationale for his Progressive Matrices (1938) which test a person's capacity to apprehend meaningless figures, see the relations between them, and by working out how each figure should be completed, "develop a systematic method of reasoning (p.1)."

A typical example is the work of the Thurstones on Primary Mental Abilities. In their two monographs (Thurstone, 1938; Thurstone & Thurstone, 1941), the concern is for statistical analysis of the sixty tests. These psychometrists admitted that "the number factor can be appraised in educational achievement tests, even though the fundamental nature of numerical thinking is not yet understood (Thurstone & Thurstone, 1941, p.5)."

Margaret Donaldson's (1963) search of the literature convinced her that test constructors have not for the most part acknowledged the need to question, for instance, the nature of the pattern process demanded by their items. "There is now a tradition in the choice of intelligence test items; but as a rule no theoretical justification can be offered (p.11)."

One exception may be Guilford (1967) who formulated a model of the structure of the intellect. He did at least recognize that the trend type of test for relation-cognition abilities is closest to the Piaget concept of seriation and can be presented in various forms, including figural (geometric) and the matrix. However these comments were limited to a single paragraph and did not pursue the nature of the process tasks involved.

The measurement specialists, nevertheless, have contributed to our knowledge of pattern, particularly with regard to inductive thinking. Using university students as his sample, Thurstone (1938) found five tests with significant saturation in the factor I (Induction). The characteristic common to these tests was that they "demanded the subject to find a rule or principle for each item in the test (p.86)." Two of these tests designated as Number Series, which had the highest loading, and Pattern Analogies involve recurrent regularity. In the 1941 study utilizing high school students, there were six tests in which the factor I appeared significantly. Of these, Letter Series, with the highest loading, and Number Patterns were of the trend type.

Pattern and the study of thinking. In contrast to the psychometrists whose efforts have been directed towards analyzing intelligence, there are other psychologists who have concerned themselves with thought and associated constructs. However, relatively few researchers have concentrated on thinking, the term itself having a connotation of activity. Among these are Frederick Bartlett, Margaret Donaldson and Jerome Bruner.

Bartlett (1958) was particularly interested in finding the most likely strategies used by adult subjects to solve serial problems. He investigated number and orientation series which required either interpolation or extrapolation. He noted that extrapolation is "usually considered to make more difficult demands than interpolation (p.34)." His own results supported this view, as did Donaldson's 1963 study in which ten-year old children tended to substitute interpolation for extrapolation when faced with alphabet problems. Thus to continue a series is likely to be harder than to fill in gaps. This evidence can be cited when the order of difficulty of pattern tasks is being conjectured.

Series extrapolation problems provided Donaldson (1963) with the data for analyzing errors in children's thinking. She listed four main variables in problems of this type as a) the nature of the terms, b) the complexity of the terms, c) the complexity of the relationships between the terms and d) the computational difficulty. She also pointed out that the end from which the series is to be extended may affect the difficulty. Because of the way books are printed in the Western world, the left-to-right movement seems to be more natural. In fact, Piaget (1969) pointed out that it is a habit which becomes irreversible. "It would necessitate a whole new learning process in order to write from right to left (p.32)." Hence to continue a series to the right can be speculated as being less difficult than the task of reversing the series to the left.

An important finding of this study pertains to the age of the succcessful subjects. Donaldson stated that in the course of the period from ten to twelve years, "these children came to understand what it means to extend a series (p.148)."

Although Jerome Bruner in his study of thinking has not specifically investigated patterns in geometric or numerical form, one particular experiment on multiple ordering is pertinent. Subjects were presented with a set of nine plastic glasses varying in three degrees of height and three degrees of diameter. When the tumblers were scattered, children of five years and older were able to restore the original configuration. The situation was repeated with a tumbler positioned by the investigator so that a reverse transposition was required. Bruner (1973) reported that the seven year old, in contrast to younger children,

> is more likely to pause, to treat the transposition as a problem, to talk to himself about "where this should go." The relation of place and size is for him a problem that requires reckoning, not simply copying (p.341).

As the number of possible errors was nine, and there were only ten subjects aged seven years, the mean number of two errors seems quite significant and indicates that even older children may have difficulty in reversing a matrix.

Even though their aim was not particularly to analyse the nature of pattern, among them, Bruner, Bartlett and Donaldson have touched on a number of aspects of the pattern process.

Pattern and the Structures of Mathematics and Cognitive Development

In the last forty years two intellectual giants have appeared. Jean Piaget has established an extensive psychological framework to explain the cognitive development of children. The other is "Nicholas Bourbaki" representing a group of eminent French mathematicians who have attempted a similar task by constructing an architecture for the universe of mathematics (Bourbaki, 1950). Piaget himself has demonstrated the parallelism between the two structural systems. The synthesis of the Bourbaki and Piagetian structures provides the construct of the pattern process.

The Mother-structures of the Bourbaki. Since 1939, the abstract nature of mathematics as a whole has been the focus of study by the anonymous Bourbaki School. With emphasis on strictly logical arrangement and insistence on the maximum generality, the group has sought the fundamental structures which are common among all the different branches of mathematics. Instead of fragmenting mathematics into the traditional fields, the Bourbaki came to the conclusion that there are three basic and irreducible structures, namely, the algebraic structure, the ordering structure and the topological structure. From these mother-structures all other topics in mathematics can be drawn by combining, differentiating or specializing the fundamental structures.

The algebraic structure can be illustrated by the operations of addition or multiplication each of which has the possibility of combining a direct operation with an inverse operation. The form of reversibility negates the operation and so produces a null product when the direct and inverse operations are performed jointly.

The pattern process exhibits this same structure; for example, a number sequence which involves successive doubling is nullified by the same sequence being halved. This process occurs whether the pattern is being continued or reversed in either direction. Similarly, for a geometric pattern where colour and shape are the properties being manipulated, continuation and reversion are complementary operations.

The second fundamental structure is the ordering structure which is based on relations. The structure imposed on real numbers by the relation of inequality is an example. $A > B$ can be transformed into $B < A$ by application of the law of duality so that the relationship between the entities remains constant. Thus the two statements can be described as reciprocal.

Patterns are sets of elements related by a common property, and obviously have an ordering structure. The feature of reprocicity is also present. The elements of a number series written from left to right express the same relationship when written in the opposite order. A matrix containing a geometric pattern based on colour and form can be reflected. Although the arrangement of the elements is different, the relationships within the pattern remain the same.

Topological structures, which form the third mother-structure of the Bourbaki scheme, are based on concepts such as proximity, continuity, continuous correspondence and limits. In both linear and matrix patterns the idea of neighbourhood is evident. One of the obvious tasks associated with pattern is the requirement to continue. Even in the absence of instructions, humans have the natural tendency to continue a pattern, just as in the physical environment. Sawyer (1955) drew attention to the close link between the notion of continuity and pattern which is "the only relatively stable thing in a changing world (p.12)."

Pattern: A Piagetian operation. Piaget's developmental theory has been summarized by Flavell (1963) and in numerous studies, for example, Cathcart (1969), Martin (1973), Reimer (1968) and Towler (1967). The four stages which have been clearly and briefly described in a number of publications by Piaget himself (1964a, 1969, 1970, 1971, 1972) are:

i. The sensory-motor, pre-verbal stage;
ii. Pre-operational representational stage;
iii. Concrete-operational stage; and
iv. Formal or hypothetic-deductive operations.

Central to Piaget's theory of the development of knowledge is his notion of an operation which is the feature distinguishing the concrete-operational stage from its predecessor.

> To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge (Piaget, 1964a, p.8).

Piaget gave examples of operations, the emphasis being on actions which enable the knower to get at the structures of the particular transformation.

> An operation would consist of joining objects in a class, to construct a classification. Or an operation would consist of ordering, or putting things in a series. Or an operation would consist of counting, or of measuring (p.8).

These descriptions and categories can be applied to the pattern process equally well. This process incorporates tasks, so there is abundant action. It is interiorized since the search for further elements in a pattern requires mental consideration of the given elements.

Another condition laid down by Piaget is that an operation must be reversible, "that is, it can take place in both directions, for instance, adding or subtracting, joining or separating (p.8)." In the operation of pattern continuing or reversing gives a parallel situation.

A third property of Piagetian operations is that they are never isolated; they are linked together into structures of increasing complexity. The pattern process, whether expressed in geometric or numerical form, by its very nature integrates classification of properties and the ordering of things in a series, both of which are named as operations. Thus the pattern process complies with all three conditions set out by Piaget to categorize an operation.

Bourbakian and Piagetian structures.

Independently of the Bourbakian movement, Piaget also reduced the individual mental structures built out of operations tc "three kinds of elementary structures in the child, corresponding to operations of classes, relations and continuous transformations (Beth & Piaget, 1966, p.186)," all of which are properties of the pattern process.

According to Piaget, the operation of classification is "in the simplest, most elementary form, an algebraic structure (Piaget, 1964b, p.35)," because the operation of the union of classes contains the negation form of reversibility, an inversion which amounts to annulling the original operation. As classification is a contributory operation within the more complex pattern process, it can be stated that pattern embodies this mental structure.

The second structure in both the Bourbakian and Piagetian schemes seems to be identical except for the label. Bourbaki's "ordering structure" and Piaget's "relations" are equivalent concepts (Piaget, 1964b; Beth & Piaget, 1966). Similarly the reciprocal form of reversibility is a property common to both structures. Piaget (1972) pointed out that the notion of transitivity with its its reciprocal property is "tied to the structuration of a series (p.10)." Thus the pattern process with its emphasis on relational order is a manifestation of both mathematical and psychological structure.

The parallelism between the Bourbakian topological structures and Piaget's third mental structure of continuous transformations is strengthened when pattern is used to illuminate the two concepts. Pattern which involves the topological properties of neighbourhood and continuity is also an example of Piaget's continuous transformations.

> All operational subject structures ... suppose a combination of production and conservation. There is always some production, that is some kind of transformation taking place. Similarly there is always some conservation, something that remains unchanged throughout the transformation. These two are absolutely inseparable. Without any transformation we have only static identity.... Without any conservation we have only constant change (1972, p.17).

In the pattern process, particularly in the numerical form, there is transformation from one element or unit of elements to the next. No pattern can exist unless transformation occurs. Similarly unless a child can keep the relationship constant, his pattern becomes a random sequence or array.

Summary. The interrelationships between mathematics, pattern and the operations of classification and conservation as representative of a psychological transformation in the child, are illustrated in Figure 1. Mathematics is concerned with number and space. Mathematical understanding, it has been pointed out, is dependent on operational conservation and classification; hence, they are shown as part of the ediface of mathematics. Pattern can be categorized as a Piagetian operation to which conservation and classification contribute. With the fusion of these psychological operations pattern is recognized not only as part of mathematics but is identified as the very core of Mathematics Education. In this key position pattern becomes the embodiment of Bourbaki's mother-structures and Piaget's corresponding mental structures.

Hierarchy of Pattern Tasks

From the literature, five pattern tasks can be abstracted. The order of difficulty is postulated from a number of transitive comparisons. In the studies on thinking by Bartlett (1958) and Donaldson (1963), interpolation was found to be easier than extrapolation or continuation. From Piagetian research on operations and reversibility, to continue a pattern from left to right is less difficult than the opposite task (Inhelder & Piaget, 1967). The order arising from these comparisons is a) interpolate, b) continue, and c) reverse. That selection should come next is based on a brief comment by Piaget and Inhelder (1973) in their study of memory and intelligence. In a test of recall, "five- to eight-year olds have greater difficulty in selecting the right model than in reconstructing it (p.377)." Although memory was the focus of the research, it seems logical to transfer this finding to the present study of pattern. The generation of a

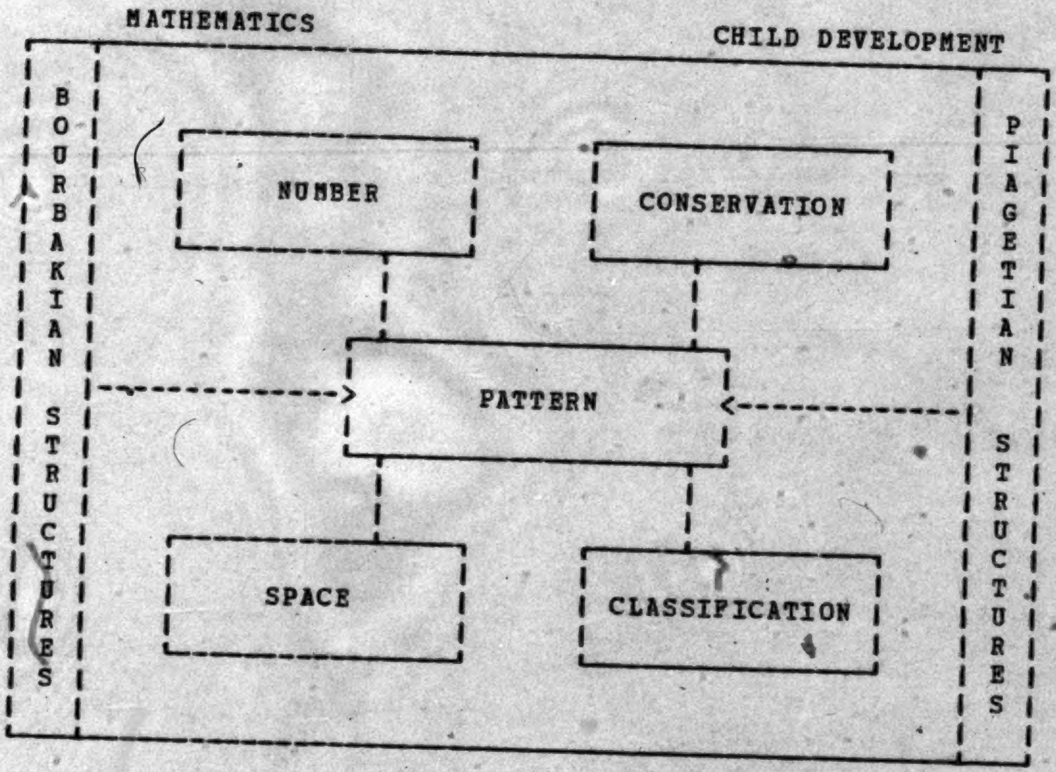
MATHEMATICS
CHILD DEVELOPMENT
BOURBAKIAN STRUCTURES
NUMBER
CONSERVATION
PATTERN
SPACE
CLASSIFICATION
PIAGETIAN STRUCTURES

FIGURE 1

PATTERN WITHIN MATHEMATICS EDUCATION

pattern requires the child to use the ability not only to devise a relationship but also to continue it. Dependent on the specific instructions, a child may generate a pattern by a reverse procedure. Hence to generate is placed last in the hierarchy of tasks.

The domains of the pattern tasks for this particular study are constrained in two ways, namely form and arrangement. Geometric form involving the properties of colour and shape, and numerical form limited to the set of whole numbers are the two selected. The order of difficulty is derived from Piaget (Beth & Piaget, 1966) who produced empirical evidence to indicate that classification by shape and colour comes earlier in a child's cognitive development than does knowledge of number.

The other constraint is in the matter of arrangement. Once again the research of Piaget and his associates (Beth & Piaget, 1966; Inhelder & Piaget, 1967; Piaget & Inhelder, 1973) is cited as grounds for stating that linear one-dimensional tasks are easier than matrix two-dimensional ones.

However, one question remains unanswered. Are matrix tasks in geometric form easier than linear tasks in numerical form? Because an understanding of number depends on operational classification, it is likely that at least the earlier geometric matrix tasks will be less difficult than any of the numerical tasks.

The hierarchy of tasks is set out in Table 1.

TABLE 1

HIERARCHY OF PATTERN TASKS

Pattern Task	Geometric		Numerical	
	Linear	Matrix	Linear	Matrix
Interpolate	A	F	K	P
Continue	B	G	L	Q
Reverse	C	H	M	R
Select	D	I	N	S
Generate	E	J	O	T

The postulated order of difficulty for each column is from top to bottom. For each arrangement the geometric form is hypothesized as being easier than its corresponding numerical form.

The order of difficulty within a specific geometric task can be drawn from the psychologists focusing on information processing research. In developing experiments to study the mechanisms of thought and the acquisition of concepts, Allen Newell, Herbert Simon and their Pittsburg associates (Klahr & Wallace, 1970; Newell & Simon, 1972; Simon & Kotovsky, 1963) have incidentally provided a considerable number of linear pattern types ordered according to the amount of difficulty experienced by young children.

PURPOSE OF THE STUDY

The purpose of this study is to examine pattern as an operational process in relation to Piaget's theory of cognitive development at the stage when abstract mathematical concepts are being formed in the child.

The specific purposes are to examine

I. the ability of Grade three pupils
    a. to interpolate,
    b. to continue,
    c. to reverse,
    d. to select, and
    e. to generate

geometric and numerical patterns.

2. the relationships, if any, of these pattern abilities to conservation of
    a. number, and
    b. area.

3. the relationships, if any, of these pattern abilities to classification:
    a. additive classes, and
    b. multiplicative classes.

4. the relationships, if any, of these pattern abilities to
    a. mathematics achievement
    b. intelligence,
    c. age, and
    d. sex.

HYPOTHESES

The following hypotheses will be tested:

1. There is no simple structure derivable from Grade three pupils' ability to
    a. interpolate,
    b. continue,
    c. reverse,
    d. select, and
    e. generate

geometric and numerical patterns when set in linear and matrix arrangement.

2. There are no significant correlations between Grade three pupils' ability to

a. interpolate,

b. continue,

c. reverse,

d. select, and

e. generate

geometric and numerical patterns, and their ability to conserve

a. number, and

b. area.

3. There are no significant correlations between Grade three pupils' ability to

a. interpolate,

b. continue,

c. reverse,

d. select, and

e. generate

geometric and numerical patterns, and their ability to classify:

a. additive classes, and

b. multiplicative classes.

4. There is no significant correlation between Grade three pupils' performance on the Pattern Processing Test and

a. mathematics achievement,

b. intelligence,

c. age, and

d. sex.

CLARIFICATION OF TERMS

Pattern: A set or series of elements related by a common property or properties which allow for interpolation or extrapolation of further elements from the same set.

Extrapolation: The extension of a series in accordance with some rule of relationship between the elements.

Interpolation: The filling of gaps left in a series by elements according to some rule of relationship.

Conservation: A specific property of an object is said to be conserved if that property remains constant despite transformation in the shape and arrangement of that object.

Transformation: The operation of changing the configuration of an object without changing the specific physical property or properties being studied.

Classification: The operation of grouping objects similar in some way into a class.

Additive Composition of Classes: The organization among a set of nested classes where each one is included in the next larger one.

$$A + A' = C \qquad A = C - A'$$

Multiplicative Composition of Classes: The partitioning of all the elements in a set on the basis of two different properties.

$$A + A' = C \qquad B + B' = C$$

Pattern Task: An activity which involves pattern and requires a subject, for instance, to interpolate or continue a series.

DESIGN OF THE STUDY

The following is an overview of the research design. A more detailed account is reported in Chapter III.

The population from which the sample was drawn consisted of the pupils belonging to 11 Grade three classrooms in four elementary schools of an urban school board. The sample of 100 was selected randomly from this population.

Instruments to investigate pattern processing and Piagetian conservation and classification were administered to each subject, either in group situations or individually.

As well, mathematics achievement, intelligence and number knowledge scores were obtained for each subject. A simple colour discrimination test was also administered.

The data were subjected to factor analysis and Pearson Product-moment correlations.

ASSUMPTIONS

It is assumed

a. that a child's response to a task is a true representation of his mental activity at that time;

b. that motivational factors will not influence significantly a pupil's performance on the various tasks.

LIMITATIONS

1. Short-term memory is obviously demanded in any task requiring cognitive effort. In the Pattern Processing Test items, the given information is always present and acts as an external memory (Newell & Simon, 1972). Hence the problem of memory is reduced though not eliminated.

2. During the administration of the Pattern

Processing Test, learning may take place and therefore the order of tasks will be held constant.

3. The selection of particular forms and arrangements are constraints on the pattern tasks and so limit the pupils' exposure to the coloured geometric shapes and the set of whole numbers.

4. Lack of knowledge of number facts may affect a child's performance on the numerical pattern tasks. As a check on this variable, a test on the number combinations corresponding to those involved in the pattern tasks was given.

SIGNIFICANCE OF THE STUDY

Traditionally Mathematics has been a school subject avoided by students when they had the choice; yet "an appreciation of mathematical pattern is fundamental to a genuine acceptance of mathematics by our pupils as relevant to their lives (Murray-Rust, 1971, p.60)." If such a claim is valid, then the study of pattern should be an integral part of every mathematics program, whether in the elementary or secondary school, or even beyond.

The prime importance of this topic may be approached from another angle. A major goal of mathematics educators is that children should enjoy, appreciate and participate in mathematical situations (Johnson, 1968). The topic of pattern lends itself to this end. However, teachers need to be convinced that the topic is more than a recreational entertainment useful for filling in free time. If pattern is to be considered a basic component in the elementary school program, knowledge of the pattern process and where it fits in a child's cognitive development become desirable. Hence the preparation of a Pattern Processing Test may provide a service to teachers who wish to learn more about the capabilities of their pupils.

OUTLINE OF THE STUDY

A review of the relevant literature will be presented in Chapter II. Chapter III contains an account of the design and administration of the instrumentation, followed by a discussion of the research procedures and an evaluation of the instruments. The results of the data analysis used to test the hypotheses are contained in Chapter IV with a categorization of errors in the children's responses to the Pattern Processing Test in Chapter V. The final chapter, Chapter VI, includes a summary and discussion of the findings as well as some implications for education and suggestions for future research.

CHAPTER II

REVIEW OF RELATED LITERATURE

The two main topics considered in this chapter are the pattern process and the Piagetian concepts of conservation and classification. A survey of the literature on the pattern process begins the chapter. In the second section a brief description of Piagetian conservation and classification is followed by a summary of the research pertaining to the particular aspects being investigated in the present study, namely conservation of number and area, and the classificatory skills of additive and multiplicative composition. A discussion on the relation of pattern to mathematical ability concludes the chapter.

THE PATTERN PROCESS

Although, over the centuries, examples of patterns have been illustrated and discussed in the literature of art and music, nature and mathematics, the pattern process has been recently brought into focus by two distinct yet related groups of researchers. Through the use of pattern sequences in their study of thinking, Frederick Bartlett and his fellow researcher Margaret Donaldson have shed light on the pattern process. Herbert Simon and his Pittsburgh associates have analysed the pattern process in conjunction with their work on artificial intelligence through computer simulation. Both groups have contributed to the terminology which helps to describe more clearly the pattern process.

Although they do not as yet form a body of literature, the isolated instances of reported research on pattern by mathematics educators provide a base for further exploration of the pattern process within the mathematics curriculum.

Studies related to the Study of Thinking

Within his major concern for the process of thinking, Bartlett (1958) gathered significant information about pattern almost incidentally. In accepting that thinking is "the use of information about something present to get somewhere else (p.24)," he faced the problem of finding more restricted forms of thinking amenable to investigation. The pattern process became central to his research with the introduction of the limiting factor of the "closed system", defined as one

> possessing a limited number of units, or items, or members, and those properties of the members which are to be used are known to begin with and do not change as the thinking proceeds (p.23).

However, "the units, items or members, may be arranged into a variety of orders or relations (p.23)." A set of specified shapes, the set of whole numbers and the English alphabet are all examples of closed systems within which patterns can occur.

In reporting his investigations on thinking, Bartlett included brief descriptions of closed systems such as a set of eight particular words or 17 specified numbers. Detailed description was provided of subjects' reactions to cryptarithmetic, a closed system in which every digit 0 to 9 has its corresponding letter and each letter is assigned a digit different from that given any other letter. The particular example of GERALD + DONALD = ROBERT was also used by the information-processors Simon and Newell (1972) to study the kinds of patterns subjects use as strategies in problem-solving situations.

Although he admitted that the process of thinking had not yet been adequately described or explained, Bartlett did isolate some dichotomous features of the pattern process. Within the context of the closed system, he distinguished between interpolation, the filling in of gaps where the terminal points are known, and extrapolation where at least one terminal point has to be satisfactorily constructed. In his consideration of either of these tasks he recognized that finding a rule inherent in a sequence is "totally different from constructing a sequence of fitting items when none of them can be immediately perceived(p.35)." In other words, the successful extraction of a rule from information given can still be followed by mistakes in the identity of the elements within the system to which the rule is to be applied. This distinction was reaffirmed by Donaldson (1963) who referred to the difference between "structure or principle and execution or application (p.135)."

Furthermore, Bartlett realized that the information available is of two kinds. Current information could be immediately perceived while stored information had to called forth from long-term memory. In order to integrate the two kinds of information, two mental processes, classsification and generalization, become involved.

> Items and groups of items making up the available evidence are specifically explored for features of agreement and difference. Those items and groups are picked out which display the greatest number of, or the most important points of agreement, and these are arranged in a sequence such that each successive step becomes empirically more probable than the preceding step (p.86).

Finally, Bartlett pointed out that the ability to find a rule lies not only within the human subject but also depends on structural features of the closed system such as regularity, symmetry and numerical grouping.

Although Bartlett was not exploring pattern as such, it did enter into his studies as one aspect of thinking. As a result he highlighted distinctive features of the pattern process.

The overall aim of Margaret Donaldson's (1963) research was the same as that of Bartlett, her particular interest being children's thinking and its contribution to our "understanding of human intellectual development(p.34)." She was concerned about the current attitude of intelligence measurers whose pronouncements she believed were not taking into account the different qualities of thinking manifested by maturing children.

Whereas Bartlett studied subjects' verbalizations mainly for the positive heuristics expressed in their problem-solving behaviour, Donaldson, using the same method of thinking aloud but in a clinical setting, categorized children's responses with emphasis on the types of errors as indicators of levels of thinking. Children of ten and twelve years of age (20 subjects per group) were presented with logical problems on two occasions two years apart. One of the problems was a series extrapolation task involving the English alphabet; hence the relevance of Donaldson's work to the present study on the pattern process even though the aim and research methodology are different.

In order to analyse the children's protocols, Donaldson distinguished between the term "error", defined as "any flaw in the thinking process", and "wrong solution", defined as "a mistaken end result (p.35)." This view is in agreement with Simon and Kotovsky's (1963) contention, reiterated by Klahr and Wallace(1970), that there is "no uniquely defined correct answer to a serial pattern extrapolation task (Klahr & Wallace, 1963, p.536)." Errors, rather than wrong solutions, were sorted into three main categories, arbitrary, structural and executive.

Arbitrary errors occurred when a subject ignored part of the available information or added to it in his search for a solution. Guessing or arbitrary allocation meant that there was no acknowledgement of the need to justify a solution, though it was conceded that no responses were the result of random guessing. On the other hand, errors of the arbitrary rule type produced some justification for the choice of the particular solution.

In Donaldson's study, no responses on the series extrapolation task were classified as arbitrary errors. A suggested explanation pointed out that this type of task is divorced from real-life and so there is less encouragement of "disloyalty to the evidence," to use Bartlett's expression. Because of the difficulty in deciding whether a child was engaged or struggling with the given, Donaldson admitted that some errors considered as flaws in structure may have been the result of ignoring or adding to the data presented.

The second category, structural errors, involved a failure to grasp the principle or logical relationship within a problem. In this category is the overlap error which results from the difficulty of conceiving correctly the inclusion of one component within another, a situation investigated by Inhelder and Piaget (1969) in their study of additive composition of classes, and a variable included in the present study of the pattern process.

In the series extrapolation task, Donaldson noted two levels of generality with regard to structural errors. The inability to grasp the general notion of what it is to extend a series according to some rule was confined to the ten year old subjects who might either generalize that one rule was applicable to any series or else interpolate when the more difficult extrapolation was required. On the other hand, the inability to discover a particular rule inherent in a specific problem dropped sharply in frequency between the ages of ten and twelve.

In certain instances of structural errors, some misunderstanding of principle was clearly involved, according to the investigator, but the exact nature of the misunderstanding was obscure.

The third category, executive errors, arose not from any failure to comprehend the relationship or structure of a problem but rather from some failure in the actual carrying out of the calculation or manipulation of the components of the problem. The executive errors found in the series extrapolation task were miscounting, reversal of pairs and difficulty with direction.

Because of her concern for the developmental trends in children's thinking, Donaldson was able to demonstrate the relativity of errors. The presence of an arbitrary error which neglects the given necessitates that a structural error which consists in failing to understand the given cannot arise. Similarly, executive errors become evident only when the structure of a problem is comprehended. As a result, children become open to structural and later executive errors only when the more primitive errors are eliminated.

Studies related to Information Processing

In their efforts to develop computer programs capable of simulating human problem-solving behaviours, Herbert Simon, Allan Newell and their Pittsburgh associates have produced considerable insight into the pattern process. Several of their research reports are highly significant to the present study.

Simon and Kotovsky (1963) used the set of letter series completion items, developed by Thurstone (1938), to investigate the congruence of a computer program and human subjects in solving the same problems, their main interest being the analysis of the differences in difficulty among problems. Their aim was to test their information processing model which consisted of a simply-symbolized vocabulary for representing serial patterns parsimoniously and a computer program which simulated the processes of sequence production and rule acquisition. The researchers first of all devised a method of pattern description which contained all the information needed to reconstruct the sequences. Within the framework of a particular alphabet, for instance, the base-ten number system or the English alphabet, the initial conditions had to be stipulated, the periodicity determined and the relations translated into a minimum number of terms such as "same" and "next". As an example, the pattern description of the sequence

aaa bbb ccc is

[M1 = Alph; a]

[M1(3), N(M1)]

Starting with a, the first letter of the English alphabet, each period produces that element (M1) three times and then replaces it by the next (N) letter of the alphabet.

In order to interpret the symbols in a pattern description, the pattern generating computer program needs the capacity to detect relations like "same" and "next" in familiar alphabets and to execute the actions signified.

On the last 15 Thurstone items, 12 adults of varied backgrounds and 67 high school seniors recorded from 35 to 75 correct responses, while the most successful version of the four computer programs was able to solve 13 of the 15 items. Another variant solved 7 of the 15 problems which could thereby be labelled as easy or hard. Except for one item, this dichotomy matched the division of the problems according to the median of difficulty for the human subjects.

Three possible reasons for the failure of the pattern generator were suggested by Simon and Kotovsky; firstly, lack of familiarity with the alphabet used in constructing a pattern; secondly, a limited repertoire of relations it can test; and thirdly, inadequate means for organizing and recording the relations it discovers into a coherent pattern description.

Problem difficulty for human subjects was predictable from the number of symbols in the pattern descriptions rather than the length of the period. Further analysis, however, showed that the presence of a problem above the median in difficulty could be determined by the need to detect and keep track of relations on two distinct lists within the sequence.

A more insidious source of failure arose when the same symbols occurred in both adjoining and corresponding positions as, for example, in the sequence urtustuttuut .

Like Simon and Kotovsky, Klahr and Wallace (1970) aimed to develop a model to explain problem-solving behaviour. Through their information analysis of two types of series completion problems, they delved further into the pattern process.

The initial test series of 24 problems involved a set of geometric shapes with two attributes, colour and orientation, while the letter series came from two different sources. The first 15 problems were originally devised by Thurstone (1938) and used by Simon and Kotovsky (1963). The second group of 24 problems was drawn up at the University of Michigan in 1965 . (Klahr & Wallace, 1970, p.249). The letter series were later translated into a number series with identical structure by the substitution of the first 26 integers for the letters of the English alphabet.

As order is not inherent in either of the attributes as they were used in the Pittsburgh study, the colour-orientaion series may be regarded as sets of non-ordered objects. The only relation between objects to be accepted or rejected is "same". On the other hand, the English alphabet has a conventional order, thus the letter series requires a greater range of relations to indicate how elements are not the same. The successful extrapolation of each of these problems depends on a pattern of relations, not just objects. Referring to the two series as Stage I and Stage II respectively, Klahr and Wallace pointed out that a Stage II computer program can operate at a Stage I level by simply reducing the number of relations involved to one: "same".

The two stages discussed by these authors are parallelled in the present study: tasks involving geometric objects with two attributes, colour and shape, are followed by number series problems which include many different relations.

From an intensive study of the protocols of five and six year old children, Klahr and Wallace found that a small set of strategies seemed to be employed by subjects in various mixes. One of these strategies involved a template building procedure whereby subjects construct templates of increasing size until they find a recurring pattern. The relational pattern determines the position in the alphabet from symbols produced immediately before, without cognizance of the relationship within a symbol pattern. A second use of the template model occurred in a backward scanning strategy in which a template of the last few elements in a series is matched against the problem. Sometimes a subject seemed to apply the template building procedure to the attributes considered as separate lists.

The other two strategies were based on "fairness"; for instance, the need for equal numbers of each element. In these cases the problems were treated as unordered collections of elements rather than a series involving order of some kind.

In devising a symbolism to represent patterns, Klahr and Wallace recognized Simon and Kotovsky's requirement that descriptions be in terms of initial values and relational patterns. They limited themselves to the four relations prior(P), same(S), next(N) and double next(T). For the sequence aaa bbb ccc the pattern description is NNN (aaa) . In this notation, the simple operator N is applied to the initial compound element aaa , the cycle being repeated indefinitely. The authors commented that all but one of the 39 letter series can be described satisfactorily if the relation "triple next" is included in the set of relations.

Ancther Pittsburgh researcher Richard Roman (1974) has developed a third method for describing patterns symbolically. More complex than either Simon and Kotovsky's or Klahr and Wallaces's notation, it can handle a greater range of patterns. There are three simple relations, count forwards (C), count backwards (BC) and repeat (R), and three compound relations, concatenate (CON), intersperse (INT) and skip (SK). AL(A) indicates that the English alphabet starting at the letter A is being used. Some examples to illustrate this notational system follow:

C(5)	5, 6, 7, 8, 9, ...
CON (C(5), AL(A))	5A, 6B, 7C, ...
INT (C(5), R(6))	5, 6, 6, 6, 7, 6, 8, 6, ...
SK (C(5), R(2))	5, 7, 9, 11, 13, ...

SK (C(5), R(x2))   5, 10, 20, 40, ...

CON(SK(C(3), R(2)); SK (AL(C), R(2))   3C, 5E, 7G, 9I, 11K, ...

Because quite different alternate descriptions can be assembled, Roman's notation seems suitable not only for computer programming but also for a mathematics curriculum which is fostering flexibility of thinking. For example, the sequence

14, 17, 22, 23, 30, 33, 38, 39, 46, 49, ...

can be described as

INT (SK(C(14), R(8)); SK (C(17), R(6,10))

or

SK (C(14), R(3,5,1,7,))

Recent research directed by Simon has focussed on the understanding of serially patterned action. Greeno and Simon (1974) differentiated three distinct tasks, namely:

> the process of discovering and inducing the pattern implicit in a sequence of numbers, letters, or other symbols;
>
> the process of representing and storing in memory the pattern that describes the sequence;
>
> and the process of using this stored information to generate the sequence from the pattern in real time (p. 187).

Through the use of the computer program, interpreter, each of the three tasks can be analysed separately and levels of difficulty postulated.

Following Restle (1970), Greeno and Simon (1974) suggested a notation using only three operators - repeat (R), transpose (Ti -add or subtract by i) and reflect (M for "mirror"). The fact that this set of operators has group properties provides a firm basis for further theoretical study of the pattern process. The authors warn that no single measure of pattern difficulty will handle all three tasks and it is unlikely that all subjects will follow a similar interpretive procedure to accomplish the sequence-generating task.

Investigations within Mathematics Education

Because of the limited reseach on pattern by mathematics educators interested in curriculum, one particular study by Cromie (1971) stands out. He investigated the development of the ability of children four to seven years of age to deal with patterns in three cognitive processes and in three cognitive modes. The processes were categorized as reproduction, identification (selection in the present study), and extension (that is, extrapolation). The modes were classified as enactive, iconic and symbolic, three phases postulated by Bruner (1960). Cromie found that the ontogeny for the processes was reproduction, identification and extension.

It is to be hoped that more studies by mathematics educators on this topic will be forthcoming.

PIAGETIAN CONSERVATION AND CLASSIFICATION

As stated earlier, Piaget's developmental theory has been summarized by Flavell (1963), and in numerous studies, (for example, Cathcart,1969; Martin, 1973; Reimer, 1968; Richardson, 1973; and Towler, 1967).

The basic framework of Piaget's theory of cognitive development is the four qualitatively different stages, sensori-motor, pre-operational, concrete operational and formal operations. A child in the pre-operational stage characteristically is swayed by perceptual cues, and is often unaware when his conclusions are self-contradictory, while in the next stage he comprehends the invariance of certain properties of an object at a conceptual level. The section of the structure relevant to this study is the dynamic and fluctuating transition from the stage of pre-operational thought to that of concrete operational thought manifested in the development of conservation and classificatory skills.

Conservation

A feature of the transitional period in the development of logical thinking is the child's gradual acquisition of the principle of conservation pertaining to quantity, number, substance, length, weight and area, designated as "first-order" conservations because they require only single applications of reversibility. Having attained this operation, the conserving child can cancel out the effects of change in order to focus on the properties of an object that have remained unchanged (Almy, 1966); he can integrate two temporally separate experiences into a single judgement (Reimer, 1968). These conservations can therefore be considered indexes of concrete operational thought (Brainerd & Allen, 1971).

In between these two stable states a child vascillates in his grasp of the notion of invariance. He may be confident of the numerical permanence of a set of blocks, yet change his mind with alacrity when faced with transformations of a set of construction bricks. Thus a child moves from an absence of conservation to necessary conservation with an intervening period in which he experiences a conflict between perceptual evidence and logical conceptions.

Research reports pertaining to Piagetian conservation have appeared in numerous books of readings such as those edited by Athey and Rubadeau (1970), Elkind and Flavell (1969), and Sigel and Hooper (1968), while explanatory interpretations have been published by Brearley and Hitchfield (1967), Farnham-Diggory (1971), Ginsburg and Opper (1969), Phillips(1969) and others. Martin (1973) contains an extensive survey of conservation research and a bibliography classified according to the particular property being explored.

The two Piagetian conservations relevant to this study are number and area. Research investigating the presence or induction of first-order conservation has concentrated on number, along with substance, length and weight, with very little being done on area. In fact, this aspect of conservation is not categorized in the _Interpretive Study of Research and Development in Elementary Mathematics_ (Suydam & Riedesel, 1969), nor in the preceding dissertation, _An Evaluation of Journal-published Research Reports on Elementary School Mathematics 1900-1965_ (Suydam, 1967).

Conservation of number. _The Child's Conception of Number_ (1965) is devoted to Piaget's investigations of various aspects of number development. He found that children perform consistently whether they are exploring conservation of continuous quantities in the form of liquids, or conservation of discontinuous quantities with containers of beads. Perceptual appearance dominates the younger children while the next stage sees subjects changing their minds even though a co-ordination of relationships is developing. Lasting equivalence appears when a child realizes that a transformation can be reversed mentally, thus ensuring conservation.

A parallel pattern resulted from the investigations of number conservation which Piaget categorized into two types of correspondence. In one case, correspondence can be provoked by external circumstances such as the association of egg with eggcup, the physical proximity of a glass by a bottle, a flower in a vase, or the dynamic exchange of one object for another. On the other hand, spontaneous correspondence is elicited through the use of similar objects and without any particular method of problem-solving being suggested.

In a typical situation structured to test correspondence, two sets of objects are paired then the objects in one set are spread out while the other group may be made more compact. The child is questioned about the relative numbers of objects in the two sets at each step during the transformations.

In making their comparisons, younger children tend to evaluate spatially, thereby ignoring the numerical relationship. At the second stage, children are capable of making the one-to-one correspondence but as soon as the visual pairing is destroyed, the quantitative equivalence no longer seems to exist. For children at the third stage, quantities remain equivalent even though the space occupied changes.

The protocols of Piaget's subjects indicate that progression through these stages of number conservation is the same for both types of correspondence; hence number conservation through provoked correspondence will be discussed here, particularly as other researchers have tended to choose this aspect for investigation.

Studies by researchers such as Dodwell (1960), Hood (1962) and Wohlwill (1962) have confirmed the general sequence of stages in number development proposed by the Genevans. An extensive review of the literature on the acquisition of number concepts can be found in Richardson (1973). According to Piaget (for example, 1971), the change in thinking revealed in success on conservation tasks occurs, on the average, at seven to eight years of age. The performance of Piaget's subjects in five different number situations is shown in Table 2.

A feature of this summary is the wide age range of children categorized at Stage II. Subjects in the transitional period between no conservation and necessary conservation are spread across a span of 24 months for the glasses and bottles situation, for the flowers and vases 16 months and for the pennies-objects exchange 26 months. The oldest subject reported was 6.11 years and he was still in Stage II in the pennies-objects exchange situation. If Piaget tested older children for number conservation, he did not include their responses in his classic reference on the topic.

Older subjects have been used in several replication studies. Almy (1970) administered Piagetian number conservation tasks to 628 second grade children (mean age 7.3 years) of whom 366 gave clearly operational performance, leaving 263 not yet conserving number. When Hyde's (1970) Bath and Doll Test, paralleling Piaget's glasses and bottles situation, was presented to six, seven and eight year old European children in Aden, 9 of the 16 eight year olds were categorized as inadequate conservers. Of the 16 children in each of the six and seven year old groups, 6 and 11 respectively were noted as non-conservers.

TABLE 2

NUMBER CONSERVATION (PROVOKED CORRESPONDENCE): AGES OF PIAGET'S (1965) SUBJECTS IN FIVE SITUATIONS

(N=43)

CORRESPONDENCE SITUATION	STAGE I	STAGE II	STAGE III
Eggs and eggcups	4.3 4.9	5.1 5.7 5.8 5.10	5.8 6.11
Flowers and vases	4.4 4.4	4.6 5.7 5.8 5.10	5.5 5.8
Glasses and bottles	4.0 4.0 5.2	4.3 4.4 4.11 5.5 5.3 5.8 5.9 5.10 6.3	5.6 6.2
Pennies-objects exchange	4.4 4.4 4.6	4.1 4.5 5.2 5.9 6.3 6.7 6.11	4.11 5.8 5.8 5.8
Pennies-objects exchange with counting aloud	3.6 3.11	5.6 5.6 6.0 6.0	6.6 6.6

Generally, the non-Europeans showed less evidence of conservation attainment.

Conservation of area. Piaget, Inhelder and Smeminska (1960) devised some ingenious problems to investigate the conservation of area. In the Barns Task two pieces of card demonstrably equal in area represent two fields in each of which a cow is pasturing. Barns of equal dimensions are in one field scattered and in the other clustered near the edge. Through this situation Piaget et al. probed a child's understanding of the invariance of area upon subtraction.

In another problem, a segment is removed from a rectangular piece of card and attached at a different spot. The child must ignore the increase in perimeter in order to acknowledge the equivalence of area.

In a variation of this type of transformation, two congruent triangular pieces made from a rectangular piece of card are rearranged into a large triangle and other shapes such as a parallelogram in order to check a child's ability to withstand the apparent change in size.

Beard (1960) paralleled the Transformed Triangles Task by the use of a form board in which rectangular and rhombic holes could be exactly filled by the same two congruent triangular wooden blocks. Six and seven year old children were asked which hole they thought was bigger and then saw the blocks fitted in. Of the 60 subjects, 23 persisted in their perceptual judgement that the rhombic hole was larger.

In a comparative study of Ghanaian and English children, Beard (1965) cut two congruent squares in half, one parallel with a side, the other diagonally. A triangular half and a rectangular half were then combined to make a "house" which was compared with the size of the original square. At age eight, both groups had a success rate of 32 per cent. At age ten, 56 per cent of the Ghanaians attained Stage III as against 62 per cent of the English children.

In their study aimed at the training of Grade one and Grade three pupils in the conservation of length and area, Beilin and Franklin (1962) used the Barns Task as well as the Rearranged Rectangles Test but failed to report the specific data. In two further experiments, Beilin (1964, 1966) used an analogue of the Rearranged Rectangles Test in the form of a Visual Pattern Board. By the insertion of prepared templates, transformations of connected or unconnected unit squares could be instantly presented, without any motion being evident. No physical manipulation was involved. Apart from their perceptual impressions, subjects could use counting in the form of iteration or translocation in making their judgements of the equivalence of area. In the 1964 study, 13 of the 26 Grade four children could not judge area equivalence correctly. The 1966 experiment was restricted to first and second grade children. The number of area conservers on the pre-test(Grade one=11,Gradetwo=28) was raised after training to 77(Grade one=27,Grade two=50) out of a total of 236 subjects equally divided between the two grades.

The Barns Task and the Transformed Triangles Task were presented to 90 Grade three pupils (mean age 8.8 years) by Blackall (1972) who found that 42 children were conserving on the Barns Task, while 30 were so classed on the Transformed Triangle Test. After analysing the rationalizations given by the subjects on the Barns Task, the investigator questioned whether this test is not so much a test of area conservation as of number conservation.

Murray (1965), who describes conservation as "a process of resisting illusion or of recognizing a disparity between appearance and reality (p.62)," presented Grades one, two and three children with illusion-distorted areas. Seventeen of the 23 Grade three children reached Piaget's

Stage III on the Delboeur concentric circles illusion. He claimed that these data support the conclusion that the transition from no conservation to necessary conservation occurs between the ages of seven and eight years. However, nearly one-quarter of this group were not operational with regard to area.

In a scalogram analysis of Piagetian area concepts, Needleman (1970) reported that the total sample of 69 boys in Grades three to eight attained the concept of area conservation. The subjects ranged in age from 8.6 to 14.4 with I Q scores from 100 to 120. However, these results are not consistent with other reported research such as Goodnow and Bethon's (1966) study in which 38 per cent of the gifted eight year olds and 56 per cent of the average eleven year olds were categorized as operational in area.

Summary. In the studies investigating number and area conservation, there is one common feature, the lack of uniformity with regard to the age of attainment of operational thinking. Nevertheless, there is agreement with Piaget on the general development of conservation concepts.

Classification

Because of the particular works selected for translation into English, the Genevans give the impression that they have concentrated on the problem of conservation so that the developmental stages are more commonly associated with that topic. However, similar stages are postulated for the development of mature classification. After the initial sensori-motor period, a child moves through the typical stages of global comparison, intuitive judgement and operational judgement (Hyde, 1970). Inhelder and Piaget who co-authored The Early Growth of Logic in the Child (1969) specify the invariant and qualitatively different stages with regard to classification as

i. graphic collections
ii. non-graphic collections
iii. hierarchical classification.

For the child between the age of two and five years approximately, the spatial arrangement plays an essential part in deciding how the elements in a collection are to be sorted; hence the term "graphic collections" to describe this level. The child may set out objects in heaps but not systematically. He often applies relations of similarity and difference to objects in successive pairs, so that small partial alignments are produced without any evidence of an understanding of the collection as a whole. He groups objects that belong together such as a square and a triangle to form a house, a tree, a fence or a horse, rather than classifying them according to common properties such as form or colour. Thus he may produce complex geometrical objects based on perceptual belonging or complex representational objects based on functional belonging.

At the level of graphic collections, the child does not comprehend the properties of classification, including the basic notion that objects with a common property form a subset.

In the subsequent stage labelled "non-graphic collections" the child is aware that each and every object has to be classified in one subset only. Because all the elements in a subset have a common property each subset is mutually exclusive. He is more flexible in being able to classify an individual object according to a number of properties. However, certain key concepts are beyond him still. He does not connect a subset with the whole collection when it removed or disassociated physically. Secondly, he does not comprehend the idea of class-inclusion even though he may intuitively structure subsets hierarchically rather than in juxtaposition. For instance, in the matter of multiplication of classes he may "construct four collections isomorphic to those of a matrix ... yet he fails to realize that the set as a whole can be split in two ways and not just one (p.168)."

According to the Genevans, what the child's thinking lacks at this time is the operation of reversibility. Although he may be quite clear that two subsets form a union, the child cannot reverse the process in order to compare a subset with the whole set. Thus he is incapable of understanding that all the elements in a subset are only part or some of the superordinate set which may be regrouped in a number of ways.

To reach the final stage of hierarchical classification, the child must acquire reversibility if he is to classify objects and operate with sets and subsets logically. It has already been noted that Piaget sees reversibility as a critical factor in a child's ability to conserve.

Included in the present study are two key classification concepts, namely additive composition of classes and multiplicative composition of classes, both involving the co-ordination of part-whole relations. Successful attainment of both these logical operations requires the ability to use the same elements in two different ways. In additive composition the child has to think simultaneously of the part and the whole, while multiplicative composition consists in classifying each element according to two different criteria simultaneously. Piaget has claimed that these two types of classification are "mastered at the same time, about the age of seven and eight years (Inhelder & Piaget, 1969, p.151)."

Additive composition of classes. In the classic Beads Test used in three Genevan investigations of additive composition (Piaget, 1965; Inhelder & Piaget, 1969; Ving Bang, 1971), the child is presented with 20 or so wooden beads, 18 of which are brown in colour with the remaining two white. The crucial question asks, "Are there more wooden beads or more brown beads?" Parallel situations involve flowers, children and coloured wooden shapes.

At the first stage of graphic collections, spontaneous classification results in ambiguous groupings. Children at Stage II recognize that the subset, primulas for instance, belongs to the class of flowers, but they cannot separate the superordinate class into two sub-classes so that it still retains its identity. At Stage III, "the whole continues to exist while its components are separated in thought (Inhelder & Piaget, 1969, p.102)."

Piaget (1965) claimed

> clear evidence of the systematic difficulty experienced by children under seven or eight in including one class in another and in understanding that a total class is wider than one that is included in it (p.166).

In his first investigation, Piaget (1965) tested 26 children mainly in the wooden beads situation. Twenty-one of these subjects ranging in age from 5.0 to 6.8 years were categorized at Stage I - the absence of additive composition; one aged 6.0 and two aged 7.2 were at Stage II while the successful operators were 6.9, 7.2 and 8.0.

Of the 69 subjects described in the later investigation (Inhelder & Piaget 1969), which favoured the flowers situation, 29 were successful at the Stage III level. The proportions of 5 out of 20 five and six year olds and 5 out of the 19 seven year olds are in marked contrast to those of the older groups in which 10 of the 17 eight year olds and 9 of the 13 nine and ten year olds were considered operational.

In another Genevan experiment conducted by Ving Bang (1971), the low proportions of 7 per cent of the five year olds and 13 per cent of the six year olds who were successful rose to 40, 60 and 70 per cent of the seven, eight and nine year olds respectively. The total numbers of subjects were not included.

These results not only provide evidence for Piaget's claim that children are seven or eight before they acquire class inclusion, but also emphasize that some children may be very much older when they reach this level of logical thinking, a condition postulated as necessary to an adequate understanding of the classes and relations of number.

Replications of the Piagetian Beads Test of additive composition , or its equivalents, have been carried out on both sides of the Atlantic. The Development of the Additive Composition of Classes is one of the Piaget Replication Studies Elkind (1961) conducted using the boys and girls situation and 25 subjects in each age group from five to eight years. At age five and six years, 12 and 14 children respectively were categorized at Stage III, while there were 19 and 23 in the two older groups.

Lovell, Mitchell and Everett (1962) presented both the wooden beads and the flowers situations to ten subjects at each age level from five to ten years. They found that no five year olds reached Stage III. Of the six, seven and eight year olds, up to 4 were successful on either tests. However, all but one of the nine to eleven year old group were able to relate the parts to one another and to the whole, thus indicating their attainment of operational thinking.

In her scalogram study of classificatory development, Kofsky (1966) used a coloured shapes situation with 20 subjects in each age group from four to nine years. (There was an additional subject in both the four and seven year old groups.) The percentage of successful children amongst the four youngest age groups fluctuated considerably from a surprising 29 per cent of the four year olds as against only 10 per cent of the six year olds. Approximately one-fifth of each of the five and seven year old groups were regarded as operational. There was a marked increase in the proportion of older subjects who reached Stage II (45 per cent of the eight year olds and 60 per cent of the nine year olds). Although Kofsky's results generally confirm Paiget's contention regarding the leap forward in operational thinking at seven or eight years, considerable numbers of older children did not pass this class inclusion test.

Almy (1970) incorporated a class inclusion task into the battery of Piagetian tests she administered to 628 Grade two pupils. Fifty of the children gave clearly operational performance on two situations involving small numbers of fruit and wooden blocks. In considering the possible interfering effect of language comprehension on the level of a child's response, Almy concluded from an associated earlier study (Miller, 1966) that "for a child lacking such insight the rephrasing of the question made little if any difference in his response (Almy, 1970, p.34)."

When Hyde (1970) administered the Beads Test to the 48 European children in her study equally divided into the six, seven and eight year olds, none of the youngest group was successful, while 7 in each of the other groups had reached Stage III. Thus slightly more than half of the older children were not yet operational.

Although primarily interested in differences in performance due to social class, Wei, Lavatelli and Jones (1971) reported that 27 of the 40 Grade two pupils (mean age 7.6 years) were operational when they were given the wooden beads task.

A summary of subjects who attained operational additive composition in the various investigations is presented in Table 3.

Although the criterion of categorization would

TABLE 3

ADDITIVE COMPOSITION OF CLASSES

PERCENTAGE OF SUBJECTS ATTAINING STAGE III IN VARIOUS INVESTIGATIONS

	Situation	N*per group	Age of subjects in years 4	5	6	7	8	9+
Elkind (1961)	children	25		48	56	76	92	
Lovell Mitchell & Everett (1962)	beads	10		0	30	40	30	100
	flowers	10		0	20	20	30	90
Kofsky (1966)	blocks	20	29	20	10	19	45	60
Almy (1970)	fruit & blocks	628				8		
Hyde (1970)	beads	16		0	44	44		
Ving Bang (1971)	?	?		7	13	40	60	70
Wei Lavatelli & Jones (1971)	beads	40		15		67		

obviously differ from study to study, the table indicates marked discrepancies among the percentages extracted from the various studies. The percentage of five year olds who have attained Stage III vary from 0 to 48 per cent, for the six year olds 10 to 56 per cent, for the seven year olds 8 to 76 per cent, and for the eight year olds 30 to 92 per cent. Even if Elkind's and Almy's studies, which show the most discrepancies, are omitted, the differences in percentage range from 20 to 48. The generalizability of such findings would need to be tentative. However, the data generally confirm the Piagetian contention of a critical change in thinking ability at seven and eight years of age.

Multiplicative composition of classes. There are two main types of multiplicative classification situations, matrix intersection completions when the subject usually selects his response from a finite set of alternatives, and spontaneous cross-classification in which the child forms his own classes from the set of objects before him. The latter method has been chosen for the present study.

In the classic Piagetian task involving free classification, there are squares and circles in two colours, red and blue, giving four sub-classes, red squares, red circles, blue circles and blue squares. After the subject has been asked to divide the objects into four piles, he is requested to make two piles first one way and then in another way. The critical response follows the instruction to make the four piles again so that the piles go together when combined horizontally as well as vertically.

An alternate situation utilizes black or white rabbits, either sitting or running; another has men, women, girls and boys, while a third situation incorporates forms of transport.

The children's responses can once again be categorized according to three stages, the first being graphic collections which indicates lack of multiplicative composition. In Stage II the child forms exhaustive subsets while still ignoring the simultaneous relationships. He may arrive at the four collections of a possible matrix, but he has no idea of cross-classification. A Stage III response is typified by immediate cross-classification, indicating the mental co-ordination of two criteria.

Amongst the researchers who have replicated the Piagetian multiplicative composition investigations are Lovell, Mitchell and Everett (1962), Rawson (1969) and Sheppard (1974).

Lovell, et al. (1962) used the rabbits situation which they presented to ten children at each age level, five to ten years. The number of successful subjects gradually increased from 0 at age five to 9 at age ten. For the age groups in between, the Stage III children numbered two, five, ten and eight respectively. Thus three out of 20 children eight or older were still not operational.

In this particular research study, Lovell and his associates administered a large battery of Piagetian classification tests to the same children and, like the Genevan school, found an increase in general operational mobility between seven and eight years of age, across a wide range of classificatory skills.

Rawson (1969) also administered a large battery of tests of cognition to 100 Grade four subjects with a mean age of 9.8 years. Included was the multiplicative composition task involving squares and circles in two colours. Sixty-four of the children were categorized as operational, leaving 36 per cent of the sample at Stage II.

A cross-multiplication test using coloured shapes and people was presented to small numbers of children at three age groups by Sheppard (1974) who found that two out of 7 six year olds were successful while 1 eight year old and 1 ten year old were not yet operational.

Summary. As with conservation of number and area, the studies replicating Piaget's research on the additive and multiplicative composition of classes have produced widely differing data on the ages of subjects attaining operational thinking. Yet, there is general confirmation of Piaget's stages in the development of classificatory skills.

PATTERN AND MATHEMATICAL ABILITY

The existence of "a special capacity or faculty underlying mathematical ability distinct from and with no essentially close connection with other forms of intellectual capacity (Brown, 1913, p.26)" has interested researchers since the turn of the century. In the early decades the evidence was not decisive, especially as Spearman, who helped to develop factor analysis, supported the primacy of general intelligence, while Thurstone (1938) identified a series of distinct multiple factors known as primary abilities. One of these was Factor I (induction) which was highly loaded by the Number Series and Pattern Analogies tests. Nevertheless, in recent years, there seems to be widespread acceptance of the group factors V (verbal), K (spatial) and N (numerical) and the general factor G (non-verbal intelligence), residual after innate general ability has been removed statistically from verbal, spatial and numerical abilities. Furthermore, researchers on both sides of the Atlantic have come to agree that an independent factor indicating mathematical ability does exist, though its status relevant to the general factor G is still a source of discussion.

Interest in mathematical ability as a group factor has now turned to the analysis of its components, one of which appears to be closely connected to the focal topic of the present study, the pattern process. The evasive methodological problem seems to be the design of tests which validly measure mathematical ability.

In the Ninth Yearbook of the National Council for Teachers of Mathematics, Hamley (1934) advocated that the central theme in mathematics education should be functionality, with its four main elements being class, order, variable and correspondence. Class and order are basic to any pattern, whether it be spatial or numerical. Hamley believed that

> mathematical ability is probably a compound of general intelligence, visual imagery, ability to perceive number and space configurations and to retain such configurations as mental patterns (1935, p.28).

Mitchell (1938) listed seven processes which might be expected to make up mathematical ability: classification, abstraction, ability to understand and use symbols and words, ordering, correspondence, operations in imagery and deduction or inference, most of which are embodied in the pattern process. In his factor analysis study, Mitchell claimed to have isolated an ordering facter beyond the predominant G factor.

Barakat's (1951) in-depth study of mathematical abilities yielded a mathematical factor which he maintained could be subdivided into two components, mechanical arithmetic which is associated with memory, and mathematical work which is closely related to the manipulation of schemes and relations. The latter component would encompass the pattern process.

Barakat's research was pursued by Wrigley (1958, 1963) whose results confirmed the usually accepted view that ability in mathematics is closely connected with general intellective capacity. His evidence also supported the contention that "apart from the influence of general ability, mathematical ability and verbal ability are independent (1958, p.75)." He concluded that a mathematical group factor exists having some relation through geometry to the spatial factor, but separate from the number factor, involving "the application of a set of highly stereotyped rules (Verncn, 1950, p.43)," and defined by speed tests of simple computation. According to Wrigley (1963), "Skill in computation has little relationship to true mathematical ability (p.48)." Unfortunately, Wrigley did not attempt to analyse the components of the mathematical factor any further.

In a recent survey on the hereditary and environmental components of quantitative reasoning, Stafford (1972) delineated three components of mathematical ability.

> The first is "arithmetic knowledge," which is basically the degree of achievement or sophist- ication in mathematics. The second is "numerical ability," which is basically skill in computational operations; and the third is "quantitative reasoning," a verbal problem-solving ability (p.183).

This scheme tends not only to ignore the general factor G but also to exclude the pattern process with its emphasis on non-verbal classes, order and relations. However, if the reviewer had extended his analysis, both the G factor and the pattern process may have appeared in a more detailed structure.

On the whole, there seems to be a case to claim that the pattern process does have connections with the mathematical ability factor.

CONCLUSION

In this chapter the present state of knowledge on the pattern process has been summarized. Researchers are gradually developing symbolic representations, descriptive vocabulary, levels of difficulty among and within tasks, strategies of solution and categorization of errors as indicators of subjects' thinking processes. The classes, order and relations inherent in the pattern process are reflected in the Piagetian concepts of conservation and classification where invariance and reversibility are complementary facets of the logical thought of the maturing child.

It is generally conceded that classes, order and relations are the bases of mathematics. Hence competency in handling them in the dynamic pattern process seems closely linked to the development of mathematical ability. For what is mathematics but a way of thought (Hull, 1970)?

CHAPTER III

INSTRUMENTATION, DESIGN OF THE STUDY AND ANALYSIS OF THE INSTRUMENTS

INSTRUMENTATION

In order to examine Grade three children's performance on the hierarchy of 20 tasks, set out in Table 1 (p.23), a Pattern Processing Test was constructed for use in this research project. The tasks selected to investigate the conservation and classification abilities of the subjects were derived from research reports published by Piaget and his associates. Number knowledge and colour discrimination were tested by simple measures while standardized tests were used to determine levels of mathematics achievement and intelligence.

A summary of the instrumentation is presented in Table 4.

TABLE 4

INSTRUMENTATION

PATTERN:

PATTERN PROCESSING TEST

Pattern Task	Geometric		Numerical	
	Linear	Matrix	Linear	Matrix
Interpolate	A	F	K	P
Continue	B	G	L	Q
Reverse	C	H	M	R
Select	D	I	N	S
Generate	E	J	O	T

CONSERVATION: Number - Linear Correspondence

Circular Correspondence

Area - Barns Task

Transformed Triangles Task

CLASSIFICATION: Additive Composition of Classes

Multiplicative Composition of Classes

STANDARDIZED TESTS: Mathematics Achievement - Canadian Test of Basic Skills

Intelligence - Canadian Lorge-Thorndike Intelligence Test

NUMBER KNOWLEDGE

COLOUR DISCRIMINATION

Pattern Processing Test

In this section, the design, administration and scoring of each of the 20 pattern tasks are outlined and discussed. A detailed description of the complete Pattern Processing Test is located in Appendix A.

The Pattern Processing Test falls into four blocks: geometric linear, geometric matrix, numerical linear and numerical matrix. Except for the generate task, each of the other four tasks within a block has a similar format and the same general basis with regard to the individual items, though slight variations are incorporated in order to minimize the practice effect that may arise from the use of identical sets of items. Any commonality within a block will be discussed first followed by the details peculiar to the individual tasks. Commonalities from block to block will be noted as they arise.

The stimulus materials for each task except the generate task were drawn, reproduced and coloured for group administration. Multiple copies of the 16 tasks with preset correct answers were laminated cardboard so that each group of subjects could work with identical task cards which remained clean and attractive thoughout the testing program. Each subject recorded his responses on the blank paper provided for each task separately, except for the numerical matrix tasks, for which the subject recorded his responses in the spaces provided on the task sheets.

As an introduction to the testing procedure of each task, the subjects were shown examples of the particular activity involved. The explanations were repeated as necessary until it was considered that each subject understood what it was he was supposed to do, a pretest training policy of importance according to Wepman (1958).

The maximum score for each task is ten. The details of the scoring procedures are explained task by task. For the purposes of this study, a response is considered correct when it is accepted as such on the consensus of three judges, the investigator and two other mathematics educators.

Geometric:Linear

Within each of the geometric tasks, both linear and matrix, the items and their order of difficulty were determined according to the 24-item test series of colour orientation problems devised by Holland (Klahr, 1974) and reported by Klahr and Wallace (1970). In the original series there were four colours and four orientations of a set of congruent shapes. For the purposes of the present study, the positions of the orientation variable were transformed into simple shapes such as square, triangle, circle, diamond, oblong, hill, wedge and ellipse with a maximum of four shapes in any one item. Pattern descriptions of the 24 items in the Holland test-series in terms of colour and shape can be found in Appendix B.

The ten items in each of the geometric linear tasks, except the generate task, were devised according to the verbal pattern descriptions set out in Table 5. The ten patterns were selected from the 24 in the Holland series, with the same order being maintained.

TABLE 5

GEOMETRIC LINEAR PATTERN DESCRIPTIONS

#1	Two elements alternating (same shape, same colour)
#2	Two alternating shape-pairs*, two colours alternating singly
#3	One shape alternating with a shape-pair (same shape, same colour)
#4	Two shape-pairs alternating (same shape, same colour)
#5	Two shapes alternating singly, two colours alternating in pairs
#6	Two elements (same shape, same colour) alternating about a constant element
#7	Two shapes alternating in threes, two colours alternating singly
#8	Two shape-pairs in two colours (same shape, same colour) alternating about one shape alternating in the same two colours
#9	Two shapes alternating singly, three colours cycling
#10	Two shapes alternating singly, three colours cycling

*Two elements of the same shape

Items #1 to #5 are each six elements long. Because of the increase in the period of the patterns involved, items #6, #7 and #8 provide nine elements. In items #9 and #10, the element to be supplied by the subject is not already present and must be inferred from the information given in five elements.

A. Geometric: Linear: Interpolate [G:L:I] In this task, the child is required to find the missing element indicated by an asterisk. No gap appears in either of the end positions.

The ten items in the G:L:I task were prepared according to the pattern descriptions set out in Table 5, with the following modifications. The gaps in items #1, #2, #6 and #7 are towards the right hand end; in items #3, #8 and #10 towards the left hand end; and in items #4, #5 and #9 in the middle.

The score is the total number of correct responses.

B. Geometric: Linear: Continue [G:L:C] In this task, the child is required to continue each pattern to the right. He records the next two elements on the blank paper placed on the right of his task card.

The ten items of the G:L:C task were prepared according to the pattern descriptions set out in Table 5, with the following modifications. Item #4, which has the same pattern as #3, finishes with an incomplete period. In item #7, the colours as well as the shapes alternate in threes.

The score is the total number of correct responses each of which consists of two elements in the right order.

C. Geometric: Linear: Reverse [G:L:R] In this task, the child is required to continue the pattern to the left. He records the next two elements on the blank paper placed on the left of his task card. In the explanation of the activity involved in reversing, the order of the two elements to be recorded was emphasized.

The ten items of the G:L:R task were prepared according to the pattern descriptions set out in Table 5, except that the patterns of items #7 and #8 were interchanged.

The score is the total number of correct responses each of which consists of two elements in the right order.

D. Geometric: Linear: Select [G:L:S] In this task, the child is required to continue the pattern to the right. He selects his response to each item from four choices of two elements each. Only the particular shapes and colours present in the information given are used in the choices.

The ten items of the G:L:S task were prepared according to the pattern descriptions set out in Table 5.

The distractors include the first pair on the left and its reverse, and the last pair on the right and its reverse, where these do not involve repetition. Where necessary, a pair of shapes from within the given pattern are coloured differently to provide alternate distractors.

The score is the total number of correct choices.

E. Geometric: Linear: Generate [G:L:G] In this task, the child is required to make four geometric linear patterns of his own choice, using squares, circles and triangles in four colours. The child places cardboard shapes on a prepared card which provides seven cells for each linear pattern. Subsequently, each child's patterns are photographed on 35mm colour film as a permanent record.

For each sequence, the child earns one point for a pattern in the shapes and one point for a pattern in the colours. The remaining two points are allocated for complexity beyond the level of alternating shapes and/or colours, or reflection around the middle element.

Geometric: Matrix

The design of the geometric tasks, except the generate task, was based on Holland's test series of linear patterns. However, the matrix form involves coordinated sets of linear sequences, not just a number of independent linear sequences. The need for the patterns to fit both ways increases the complexity of the relations and also requires that the pattern descriptions account for the vertical as well as the horizontal arrangement.

The order of difficulty also affects the complexity of the patterns. The length of the period is a difficulty factor in linear patterns but a different situation exists for matrix tasks. In a four by four matrix there are 16 elements already; therefore, if the size of the matrix is to be kept within reasonable limits, the complexity of the relations becomes the important difficulty factor.

F. Geometric: Matrix: Interpolate [G:M:I] The format for this task is a four by four matrix from which the middle four cells have been removed. The child is required to supply the missing elements so that there is a pattern going both ways.

The five items in the G:M:I task were prepared according to the following pattern descriptions:

#1	Two elements alternating (same shape, same colour)
#2	Two shape-pairs alternating horizontally, (same shape, same colour)
#3	Two shapes alternating, four colours cycling horizontally and alternating in pairs vertically
#4	Two shape-pairs alternating vertically, four colours cycling horizontally and alternating in pairs vertically
#5	One shape alternating with two shapes horizontally and vertically, four colours cycling vertically and alternating horizontally

The child records his responses on a sheet of paper placed under each of the task cards.

Within each item, the child earns a point if the colour in all cells is correct and a point if the shape in all cells is correct. The score is the total number of points.

G. Geometric: Matrix: Continue [G:M:C] In this task, the child is required to supply the next element following each cell on the right and along the bottom of a four by four matrix so that the pattern goes both ways.

The five items in the G:M:C task were prepared according to the following pattern descriptions:

#1 Two elements alternating (same shape, same colour)

#2 Two shape-pairs alternating horizontally, two colours alternating

#3 Two shapes alternating, four colours alternating in pairs horizontally

#4 Two shape-pairs alternating vertically, four colours cycling horizontally, and alternating in pairs vertically

#5 One shape alternating with two shapes horizontally and vertically, four colours cycling vertically and alternating horizontally

Each vertical or horizontal line of four correct responses earns one point. The score is the total number of points.

H. Geometric: Matrix: Reverse [G:M:R] In this task, the child is required to supply the next element following each cell on the left and along the top of a four by four matrix, so that a pattern goes both ways.

The five items of the G:M:R task were prepared according to the following pattern descriptions:

#1 Two shapes and two colours alternating in pairs

#2 Two shape-pairs alternating,
two colours alternating

#3 Three shapes cycling vertically and horizontally,
two colours cycling in pairs, separated by
two other colours alternating

#4 Two pairs of shapes alternating,
one shape retaining one colour,
the other three colours cycling

#5 Similar to #4 with different shapes and colours

Each horizontal or vertical line of four correct responses earns one point. The score is the total number of points.

I. Geometric: Matrix: Select [G:M:S] In this task, the child is required to continue the pattern one element in each cell to the right and along the bottom of a four by four matrix. He selects the responses from four choices which use only the particular shapes and colours present in the information given.

The five items of the G:M:S task were prepared according to the following pattern descriptions:

#1 Two elements alternating (same shape, same colour)

#2 Two shape-pairs alternating, two colours alternating in pairs

#3 Two shape-pairs alternating, two colours alternating singly

#4 Two shapes alternating, four colours cycling vertically and horizontally

#5 Three shapes cycling vertically and horizontally, three colours cycling horizontally, with two colours cycling in pairs separated by two other colours alternating

The distractors include the first element and the last element of the particular row or column being extended, and either the first or last shape in another colour. Where necessary, the remaining choices are elements from the particular row or column.

Each vertical or horizontal line of four correct responses earns one point. The score is the total number of points.

J. Geometric: Matrix: Generate [G:M:G] In this task, the child is required to make two matrix patterns of his own choice, using squares, circles and triangles in four colours. The child places the cardboard shapes for each pattern on a prepared card bearing a five by five matrix diagram. Subsequently, each child's patterns are photographed on 35mm colour film as a permanent record.

The scoring procedure takes into account the three transformations of reflection, translation and rotation. For each of the two matrix patterns, two points are earned if there is a pattern in either the shape or colour, in the horizontal or vertical directions. If there is only one detrimental element, one point is earned. The remaining two points for each matrix are allocated for complexity in the form of three or more shapes or colours. The score is half the number of points.

Numerical: Linear

In order to establish a set of numerical sequences with an order of difficulty, pattern descriptions were prepared of the items in the number series sections of the following tests: SRA's *Short Test of Educational Ability*, the *Kulmann-Anderson Test*, the *California Short-form Test of Mental Maturity*, the *Laycock Mental Ability Test*, the *Terman Group Test of Mental Ability*, and Cattell's *Culture Fair Test of g*.

As a result of meshing the descriptions, a set of numerical relations with an order of difficulty emerged, though many more details would need to be considered before a precise order of difficulty could be claimed. The pattern types formulated are set out in Table 6. It may be noted that in this study the term "multiple," though technically inaccurate, has been retained when constant addends occur from a non-zero origin in order to emphasize the commonalities between the two pattern types.

Another term, "addend," is given a specific meaning within this study. It refers to the number which is being added to the previous element of a sequence.

3

TABLE 6

NUMERICAL LINEAR PATTERN TYPES

#1	Forwards and backwards by 1s.
#2	Multiples of 2, 3, 4, 5, and 10 a. forwards b. backwards c. about a constant
#3	Multiples of 2, 3, 4, 5, and 10 from non-zero a. forwards b. backwards c. about a constant
#4	Alternating addends of 1 to 3
#5	Triads of consecutive numbers a. constant addend between triads b. about a constant
#6	Multiples of 6, 7, 8 and 9 a. forwards b. backwards c. about a constant
#7	Multiples of 6, 7, 8 and 9 from non-zero a. forwards b. backwards c. about a constant
#8	Alternating sequences
#9	Increasing addends and differences
#10	Decreasing addends and differences
#11	Alternating multiples
#12	Doubling
#13	Addend of 1 to 3 coupled with doubling
#14	Halving

The ten items in each of the numerical linear tasks, except the generate task, were drawn from this set with the same general order being maintained. The six numerals of each sequence were written in one of four colours on two sheets of yellow paper which were then attached to the front and back of 12cm by 22cm task cards and laminated.

K. Numerical: Linear: Interpolate [N:L:I] In this task, the child is required to find for each sequence the missing element indicated by a hole in the task card. The child's response is written on a piece of paper placed under the task card. The gaps vary between the second and fifth positions.

The ten items in the N:L:I task were drawn from the pattern types set out in Table 6 and their pattern descriptions follow below. The bracketed numeral indicates the initial condition or starting point of the sequence.

#1	Backwards by 1 [14]
#2	Multiples of 3 [3]
#3	Multiples of 5, non-zero [3]
#4	Alternating addends of 1 and 2 [5]
#5	Multiples of 6, backwards [36]
#6	Alternating multiples of 4 and 3 [4, 3]
#7	Increasing addends [3]

#8 Multiples of 4, backwards, non-zero [23]
#9 Decreasing differences [26, 20]
#10 Doubling [1]

The score is the total number of correct responses.

L. Numerical: Linear: Continue [N:L:C] In this task, the child is required to continue each pattern to the right. He records the next two elements on a piece of paper placed to the right of his task card.

The ten items of the N:L:C task were drawn from the pattern types set out in Table 6 and their pattern descriptions follow below. The bracketed numeral indicates the initial condition or starting point of the sequence.

#1 Multiples of 5 [20]
#2 Multiples of 2, backwards, non-zero [21]
#3 Multiples of 6 [6]
#4 Alternating addends of 1 and 2 [2]
#5 Multiples of four, backwards [36]
#6 Alternating addends of 1 and 3 [4]
#7 Increasing addends [2]
#8 Multiples of 5, backwards, non-zero [47]
#9 Alternating multiples of 3 and 4 [3, 4]

#10 Doubling [1]

The score is the total number of correct responses each of which consists of two elements in the right order.

M. Numerical: Linear: Reverse [N:L:R] In this task, the child is required to continue the pattern to the left. He records the next two elements on a piece of paper placed to the left of his task card.

The ten items of the N:L:R task were drawn from the numerical pattern types set out in Table 6 and their pattern descriptions follow below. The bracketed numeral indicates the initial condition or starting point of the sequence.

#1	Multiples of 2 [8]
#2	Multiples of 10 [40]
#3	Multiples of 4, non-zero [13]
#4	Alternating addends of 1 and 2 [7]
#5	Multiples of 3, backwards [24]
#6	Alternating multiples of 10 and 2 [80, 12]
#7	Decreasing differences [20, 15]
#8	Multiples of 6, backwards, non-zero [35]
#9	Alternating multiples of 6 and 4, Backwards [18, 12]
#10	Doubling [16]

In the explanation of the activity involved in reversibility, the order of the two elements to be recorded was emphasized.

The score is the total number of correct responses each of which consists of two elements in the right order.

N. Numerical: Linear: Select [N:L:S] In this task, the child is required to continue the pattern to the right. He selects his response to each item from four choices.

The ten items of the N:L:S task were drawn from the numerical pattern types listed in Table 6. Their pattern descriptions are set out below. The bracketed numerals indicate the initial condition or starting point of the sequence.

#1	Multiples of 11 [11]
#2	Multiples of 6 [12]
#3	Triads, subtracting 3 [18]
#4	Multiples of 5, non-zero, alternating with a constant [4, 8]
#5	Multiples of 3, non-zero [2]
#6	Backwards by 1s [9], forwards by 1s [1], separated by a constant [8]
#7	Alternating addends of 2 and 3 [4]
#8	Multiples of 9, backwards, non-zero [66]

#9 Increasing addends [8, 10]

#10 Addend of 1, then double

The distractors include the numbers after and before the final element, the decade number after the final or penultimate element, and a multiple associated with the final or the penultimate element.

The score is the total number of correct choices.

O. Numerical: Linear: Generate [N:L:G] In this task, the child is required to make five numerical linear patterns of his own choice. He writes at least six elements of each sequence on a blank piece of paper.

In the scoring procedure, the two points allowed for each of the five linear patterns are allocated according to the following scheme:

0 points:
- a. A repeated sequence
- b. Attempt but i. no pattern obvious
  - ii. insufficient information
- c. Not attempted

1 point:
- a. Sequence of multiples which would originate at 0
- b. Odd numbers
- c. The relationship(s) in one complex sequence repeated in another sequence, for example,

1, 2, 4, 5, 7, 8

3, 4, 6, 7, 9, 10

d. Visual patterns
e. Incomplete complex patterns

2 points: a. A complex sequence, for example, multiples not originating at 0
b. Alternating sequences

Numerical: Matrix

The design of all the numerical matrix tasks, except the generate task, was drawn from the numerical pattern types set out in Table 6. As with the geometric matrix tasks, the need for the patterns to fit both ways increases the complexity of the problems. The child's responses in each of the first four numerical matrix tasks are written directly on the task sheet.

P. Numerical: Matrix: Interpolate [N:M:I] In each task, the child is required to fill four gaps in a three by three matrix. In each of the items, the top left hand and bottom right hand elements are given, while the blank cells vary in position, thereby adding a difficulty dimension. The pattern descriptions are listed below.

	*Initial Condition	Horizontal	Vertical
#1	2	Multiples of 2	Multiples of 2
#2	5	Multiples of 5	Multiples of 2, non-zero

#3	10	Multiples of 10	Multiples of 3, non-zero
#4	20	Multiples of 3	Multiples of 2, backwards
#5	8	Multiples of 3, non-zero	Multiples of 5, non-zero

*Top left hand cell

In each item, a child scores two points if the four elements are correct, and one point if two or three elements are correct. The score is the total number of points.

Q. Numerical: Matrix: Continue [N:M:C] In this task, the child is required to supply the next element following each cell on the right and along the bottom of a three by three matrix so that the pattern goes both ways.

The five items of the N:M:C task were drawn from the numerical pattern types set out in Table 6 and are presented below.

	*Initial Condition	Horizontal	Vertical
#1	2	Multiples of 2	Forwards by 1s
#2	6	Multiples of 6	Multiples of 4, non-zero

#3	23	Multiples of 4, non-zero	Multiples of 4, non-zero, backwards
#4	26	Multiples of 3, non-zero, backwards	Multiples of 4, non-zero, backwards
#5	3	Forwards by 1s	Doubling

*Top left hand cell

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

R. Numerical: Matrix: Reverse [N:M:R] In this task, the child is required to supply the next element following each cell on the left and along the top of a three by three matrix so that a pattern goes both ways.

The five items of the N:M:R task were drawn from the numerical pattern types listed in Table 6 and are set out below.

	*Initial Condition	Horizontal	Vertical
#1	6	Multiples of 2	Multiples of 3
#2	15	Multiples of 2, non-zero	Multiples of 3, backwards
#3	23	Multiples of 2,	Multiples of 6,

		non-zero	backwards
#4	13	Multiples of 6, non-zero	Multiples of 4, non-zero
#5	8	Doubling	Multiples of 2, backwards

*Top left hand cell

Each horizontal or vertical line of four correct responses earns one point. The score is the total number of points.

S. Numerical: Matrix: Select [N:M:S] In this task, the child is required to continue the pattern both ways by selecting one of the four choices provided in each cell to the right and along the bottom of a three by three matrix.

The pattern descriptions of the five items are set out below.

	*Initial Condition	Horizontal	Vertical
#1	1	Forwards by 1s	Multiples of 2, non-zero
#2	3	Multiples of 3	Multiples of 2, non-zero
#3	6	Multiples of 5, non-zero	Multiples of 6

#4	13	Multiples of 4, non-zero,	Multiples of 3, non-zero backwards
#5	5	Doubling	Multiples of 2, non-zero

*Top left hand cell

The distractors include the next number and the decade number after the final element in the right hand and bottom cells and multiples associated with these final elements. Where necessary, alternate distractors were provided by adding together the final two elements.

Each vertical or horizontal line of four correct responses earns one point. The score is the total number of points.

T. Numerical: Matrix: Generate [N:M:G] In this task, the child is required to generate two numerical matrix patterns of his own choice. He is provided with a sheet of paper bearing two blank four by four matrix grids.

The scoring procedure allows eight points for each matrix according to the following scheme:

1 point: a. Each row and column with a recognizable pattern

b.
16 cells filled by the 1 to 16 sequence
c.
One line repeated four times

If each line of one direction is a set of consecutive numbers, two points only are earned, with one point if any one line is not correct. If another sequence appears in the other direction, one further point is recorded.

Points are earned for one direction only if one consecutive sequence is used or if the pattern is identical both ways. If the matrix is correct, one further point is recorded.

Two additional points are allocated for complexity.

1 point: a.
A matrix based on a line of 1s
but different patterns generated
b.
Sequences in reverse order

2 points: a.
Different patterns across
and down, neither containing a line of 1s
b.
Doubling

The score is half the number of points.

Piagetian Conservation and Classification

The conservation and classification situations explored during the present investigation were all derived from tasks described by Piaget and his associates (Inhelder & Piaget, 1969; Piaget, 1965; Piaget & Inhelder, 1967;

Piaget, Inhelder & Szeminska, 1960).

The testing procedure incorporated the Piagetian method of observing and questioning children individually. Although the Piagetian method of questioning is not a standard technique, the type of question asked to elicit the conservation response is not a significant source of variance (Cathcart, 1969; Mermelstein, 1967; Mermelstein & Shulman, 1967; Pratoomaj & Johnson, 1966).

In the present study considerable flexibility was employed in order to ensure that the children understood the verbal meaning of the questions. Neutral but supportive comments were made by the investigator in order to encourage the child to continue his active participation in the test situation.

In each of the respective tasks, the subjects were categorized as conservers, partial conservers or nonconservers, or as classifiers, partial classifiers or nonclassifiers. If a child's responses indicated a change of level of thinking from one stage to another during the testing process, the subject was categorized as partial conserver or a partial classifier. For the purposes of later statistical analyses, conservers and classifiers were given a numerical label of three, the partial conservers and partial classifiers two, and nonconservers and nonclassifiers one.

Conservation:

Conservation of number and area were each tested in two situations with each child individually. The number tasks were derived from Piaget (1965) while the area tasks taken from Piaget and Inhelder (1967) were the same as those used by Blackall (1972).

During the task administration, questions eliciting rationalizations were varied, for example, "What makes you think so?" "How come?" "How do you know?" "Why?" After each transformation, the child was asked the relevant criterion question, for instance in the Linear Correspondence Task,

> Are there as many squares as circles or are there more squares or are there more circles?

The order of choices implied in the questions was varied to avoid perseveration.

Conservation of number.

Item 1. Linear Correspondence Task. In his investigations of number conservation, Piaget (1965) included situations involving materials such as counters and pennies. Objects of a similar nature in the form of circular and square plastic shapes were used in the present study to test the provoked correspondence aspect of number conservation. The number of objects, however, differed from Piaget's reported maximum of ten pairs of objects, though the usual quantity was five to seven pairs. In the present investigation fourteen shapes were manipulated. Such a number was chosen to ensure that a Grade three child was not depending on his grouping experiences nor on his ability to count rapidly in order to make a correct judgement.

The testing procedure began with the subject helping the investigator to set up a one to one correspondence between the squares and circles. The criterion question was

> Are there as many circles as there are
> squares or are there more circles or are
> there more squares?

As mentioned earlier, the order within the question was varied to minimize a child responding to expectation or a particular linguistic position.

The circles were then stacked into three piles. To check on his understanding of the language of the criterion question, the child was asked

> How many circles do you think there are?
> 10? 15? 3? 20?

The squares were spread out and the criterion question asked once more, but in a different order. The criterion question was repeated after the squares were piled and the circles spread out.

If a child's response indicated that he was a nonconserver, a further instruction was given while the investigator spread out her hands to receive the plastic shapes.

> Give me as many circles (or squares) as you think there are squares (or circles).

A subject was categorized as a conserver if he answered the criterion question correctly on the three occasions, as partial conserver if he answered correctly on one of the occasions when the shapes were not in perceptual correspondence, and as nonconserver if he answered correctly only when the shapes were in perceptual correspondence.

Item 2. Circular Correspondence Task. The testing situation and categorization procedure of the Circular Correspondence Task were identical to those of the Linear Correspondence items except for two conditions. Firstly, 28 "fun" animals identified as green or purple provided the stimuli. Secondly, the arrangement of the objects was circular rather than linear.

Conservation of area.

Item 1. The Barns Task. This task replicated Piaget's investigation into a child's understanding of the invariance of an area relationship upon subtraction of equal regions (Piaget *et al.*, 1960). Two pieces demonstrably equal in area represented two fields in each of which a cow is grazing. Wooden blocks (2cm cubes) represented barns which reduced the amount of pasture available to the cows.

Barns were placed on each field one at a time so that the number of barns was always equal. The criterion question was

> Does this cow have as much to eat as that one, or does this one have more to eat or does that one have more to eat?

The investigator gestured appropriately when the criterion question was asked after the addition of each pair of barns.

The investigator checked with the subject that the two pieces of card were the same size and, therefore, that the two cows each had the same amount to eat. Initially both sets of five barns were scattered; then the barns in one field were positioned contiguously along the edge of the field. If a subject rejected the equality of area when the barns were in the contiguous and scattered positions, the two sets of blocks were both positioned contiguously according to the same arrangement. One set of barns was then scattered as before.

A subject was regarded as a conserver only if he could explain the equivalence of the two pastures when eight pairs of barns had been placed in the contiguous and scattered positions. Similarly, the partial conserver as well as the nonconserver was judged on his response when the barns were in the contiguous and scattered positions. A child who answered correctly at some time during the task administration was noted as a partial conserver, while the nonconserver was one who rejected the equivalence of area when the barns were arranged in the contiguous and scattered positions.

Item 2. Transformed Triangles Task. This task was also a replication of a Piagetian investigation into the conservation of area upon transformation. To acknowledge the equality of area, the child must ignore the increase in perimeter (Piaget et al., 1960). Two congruent triangles made from a rectangular piece of card were arranged into a large triangle as well as other shapes such as a parallelogram in order to examine a child's ability to withstand the apparent change in size.

In the present study, the materials used were wooden blocks (a 15cm square, two congruent right-angled triangular blocks which together form a 15cm square, two oblongs 15cm long and 7^1/2 cm wide).

The subject was presented with the two triangles arranged as a square - "your backyard." The investigator had the square - "my backyard." The two triangles were formed into a larger right-angled triangle. The criterion question was

> Do you have as much room in your backyard as I have in mine, or do you have more or do I have more?

If a subject's answer indicated that the regions were different in area, a second question was asked.

> If you have to plant your backyard with grass and mine with grass, would you need as much grass for yours, or for mine, or would they both need just as much?

The triangular blocks were arranged as a large triangle and then as a parallelogram. The oblongs in the form of a square represented the backyard belonging to the subject's teacher who was referred to by name. The oblongs were compared with the square and then with the triangles in various positions.

The children were categorized as conservers, partial conservers and nonconservers. The child who acknowledged the area equivalence in each arrangement of the blocks was categorized as a conserver. The partial conserver answered correctly sometimes, while the child who maintained there was a change of area with the first rearrangement of the blocks was categorized as a nonconserver.

Classification:

Tasks to explore a child's ability to classify additively and multiplicatively replicated two situations described by Piaget (1965) and Inhelder and Piaget (1969). The testing procedure followed Whyte's (1971) Test Schedule: Piagetian Tasks which attempts to standardize the Genevan protocols.

The subject was asked to explain his answers throughout the testing session.

Item 1. Additive Composition of Classes. In the Piagetian exploration of additive composition of classes described extensively in the 1965 report and less so in the 1969 publication, various materials were used, although the situation has become known as the Wooden Beads Test. Whyte

(1971) uses plastic beads on the grounds of their availability. For the same reason 18 yellow and 2 green plastic tiles, 2cm square, were chosen for the present study.

In the pre-test training session, the child handled the materials and was questioned in order to check that he understood that all the tiles were plastic, some being yellow, the others being green.

When the part was being compared to the whole in the testing situation proper, the criterion question was

> If I made a row with all the plastic tiles, and if I made a row with all the yellow tiles, which row would be longer?

During the session, the unsuccessful subject's attention was drawn to the plastic nature of the green tiles. He was given the opportunity to arrange the tiles in order to compare the two rows. He was finally asked

> Are there more plastic tiles or more yellow tiles?

The child was categorized as a classifier if he correctly answered the criterion question immediately as an indication of his understanding of the relationship of class and subclass. The partial classifiers were those who answered correctly on less than all occasions. The child who remained firm that the part was greater than the whole was categorized as a nonclassifier.

Item 2. Multiplicative Composition of Classes. Inhelder and Piaget (1969) used a number of different situations to determine whether a child realizes that subclasses which have been separated can be reunited. In the present study, the subject was required to sort "16 geometrical objects consisting of 4 blue squares, 4 red squares, 4 blue circles and 4 red circles (p.165)" in three ways, spontaneously, into two subsets and into four subsets. The child was then asked the criterion question

> Can you put these [for example, the blue squares] with these [the red squares] ?

About each of the pairings involving colour and shape. As in the other Piagetian tasks, the child was asked to explain his choice of action.

A child who recognized that the four subsets could be regrouped into two subsets both ways was categorized as a classifier. A child who admitted that the four subsets could be regrouped into two subsets in one way but not the other was categorized as a partial classifier. A child who maintained that the four subsets could not be regrouped was categorized as a nonclassifier.

Standardized Tests

Mathematics Achievement

Of the eight subtests in the Canadian Test of Basic Skills, two are concerned with mathematics achievement. The first, Mathematical Concepts, was designed to evaluate the students' understanding of the number system, mathematical laws and the operators. The second subtest was claimed to measure problem solving ability. A tally of the two subscores furnishes a comprehensive mathematics score for each child.

The two subtests were administered and scored according to the procedures set out in the authors' manual.

Intelligence

The Canadian Lorge-Thorndike Intelligence Test was designed to measure intelligence described by the authors as "the ability to work with ideas and relationships among ideas (Wright, Thorndike & Hagen, 1972, p.1)." At the elementary school level there are two batteries, verbal and nonverbal. The verbal battery includes five subtests: vocabulary, sentence completion, arithmetical reasoning, verbal classification and verbal analogies. The three subtests of the nonverbal battery are figure analogies, number series and figure classification. The two subscores added together produce a composite intelligence score.

The tests (Level A) were administered and scored according to the procedures set out in the authors' manual.

Other Tests

Number Knowledge

As half the tasks in the Pattern Processing Test are numerical, a simple test of number knowledge was devised and administered. Spaces were provided on prepared sheets for the children to fill in the multiples of 2, 3, 4, 5, 6, 8 and 10. In order to minimize any reading difficulties the words "count on," which appear on the sheet, were read aloud to the children. A copy of the sheet can be found in Appendix C.

Two points were allocated to each correct sequence of 3s, 4s, 6s, and 8s, with the remaining two points being shared by the multiples of 3, 5 and 10. One incorrect addition earned one of the two points; more than one incorrect addition, no points. The score is the total number of points.

The children were encouraged to use whatever method, including finger counting, they wished in order to complete the sequences. Although no time limit was set, only a few children needed more than seven minutes to complete this test.

Colour Discrimination

The four colours used in the geometric tasks of the Pattern Processing Test were red (R), green (G), blue (B) and yellow (Y). In case colour blindness or deficiency should adversely affect a child's performance, a simple colour discrimination test was administered.

The coloured cardboard shapes used by the children to generate patterns [G:L:G and G:M:G] were mixed in individual boxes. The children were asked to put all the shapes of the same colour into heaps. The investigator then asked each subject individually to name the colours describing his heaps.

RESEARCH PROCEDURES

Sample

The Edmonton Public School Board was requested to allocate four schools each having at least two Grade three classes, with the provision that no adaptation classes of specially chosen pupils be included. Eleven classes were subsequently made available to the investigator. The table of random numbers in Kerlinger (1973) was used to randomly select 100 of these Grade three pupils, no restriction being placed on children with low intelligence or physical handicap. Although sex was not a stratification variable, the sample fortuitously contained fifty girls and fifty boys.

The sample ranged in age from 92 to 134 months, with a mean of 104.5 months and a standard deviation of 5.5 months. The sample's intelligence scores derived from the Canadian Lorge-Thorndike Intelligence Test, ranged from 73 to 139, with a mean of 114.0 and a standard deviation of 14.4.

Pilot Study

A class of 20 Grade three pupils with equal numbers of girls and boys provided a group for pilot study purposes. Ten Grade four pupils were also included. The purposes of the pilot study were fourfold: first, to obtain a measure of reliability for the Pattern Processing Test. In order to do so, a parallel form of the Pattern Processing Test (Form B) was prepared. The items of Form B were based on the same pattern descriptions and types as the main form (Form "A") although different colours, shapes and numbers appeared on the task cards. Half of the 30 children did Form A followed by Form B, while the other half reversed the order of the two forms. The results of the Pearson Product moment correlation analysis are presented and discussed later under "Evaluation of the Instruments."

The second purpose of the pilot study was to consider the adequacy of the number knowledge test as a simple check on a child's facility with number. The findings from this test are also discussed under "Evaluation of the Instruments."

The third purpose was to determine whether conservation of number should be included in the main study. Doubt about the ability of all Grade three pupils to conserve number arose from Blackall's (1972) study. The Linear Correspondence Task, described above, was presented to each child individually. Of the 20 Grade three subjects, 12 were categorized as conservers, 2 as partial conservers and 6 as nonconservers. Among the Grade four subjects, one was categorized as a partial conserver and three as nonconservers. These results were the basis of the decision to include conservation of number tasks in the main study instrumentation.

The fourth purpose of the pilot study was to assess time limits and to allow trial administration of the Pattern Processing Test in a group situation. The time limits resulting from the administration of the two forms have been incorporated in the description of each task set out in Appendix A.

Data Collection

The data were collected between January and April, 1974. The classroom teachers, all of whom hold current Alberta teaching certificates, administered the Canadian Lorge-Thorndike Intelligence Test in January and the mathematics sections of the Canadian Test of Basic Skills in February. The intelligence tests were optically scored on the school board's computer, while a graduand in elementary education scored the two mathematics subtests. The procedures set out in the authors' manuals for the two standardized tests were followed. The number knowledge test was also given by the classroom teachers in February and scored by the investigator.

During March and April the Pattern Processing Test was administered to the 100 subjects in the sample in five groups by the investigator who corrected and scored the children's responses. Each group attempted the tasks in the same order. The colour discrimination task was incorporated in the Pattern Processing Test sessions. During the same months the administration of the conservation and classification tasks was shared between a graduate student in educational psychology and the investigator who categorized the children's responses. The order of the six Piagetian items was varied but not to a particular pattern.

For the group and individual testing sessions, each school provided comfortable, well-lit and undisturbed quarters.

Age and intelligence scores were secured from the cumulative record cards.

Analysis of Data:

The data for this study were analysed using the following procedures which have been prepared as computer programs by the Division of Educational Research of the University of Alberta.

1. Factor Analysis (FACT01)

   A correlation matrix leading to a factor analysis, a technique used to determine if the original set of variables can be reduced to a set of factors amenable to interpretation.

2. Pearson-Product Moment Correlation (DEST02)

   A correlation matrix computed for demographic, personal and instrumentation variables over the total sample.

EVALUATION OF THE INSTRUMENTS

A detailed description of the design, administration and scoring of each of the instruments used in this study has been presented earlier. In this section an evaluation of these instruments, based mainly on the data from the present study, is reported.

Pattern Processing Test

The Pattern Processing Test will be evaluated from three aspects: first, validity; secondly, reliability; and thirdly, the order of difficulty within and between the separate tasks.

Validity. The aspect of validity relevant to the present study is construct validity, the analysis of the meaning of test components in terms of psychological concepts (Cronbach, 1970) which are "constructed" to summarize or account for the regularities or relationships observed in behaviour (Thorndike & Hagen, 1969).

The construct under consideration in the present study is pattern processing ability. The validation of the instruments used to measure this construct depends on the closed matrix established among the five pattern processes, the two forms (geometric and numerical) and the two arrangements (linear and matrix). Each cell in the task matrix contains a test incorporating the three components of pattern processing, process, form and arrangement, in all possible combinations.

The five pattern activities were extracted from investigations conducted by Bartlett (1958), Donaldson (1963), Klahr and Wallace, (1970) Simon and Kotovsky (1963) and others. The two forms are commonly found in elementary mathematics programs, while the two arrangements, which Piaget and his Genevan associates have explored, are found in more advanced mathematics programs.

The results of the factor analysis applied to the children's performance on the Pattern Processing Test (reported in Chapter IV) lend support to the validity of pattern processing ability as a construct.

Reliability. One of the purposes of the pilot study was to obtain a measure of reliability for the Pattern Processing Test. A parallel form (Form B) was prepared according to the same pattern descriptions and types as the main form. The means and standard deviations of the parallel forms are shown in Table 7. Product Moment correlation coefficients between the two forms were computed using total task scores. The correlation coefficients and associated probabilities are presented in Table 8.

Twelve of the tasks had a high parallel forms reliability ($p<.01$) while three had moderate reliability ($p<.05$), with another one approaching this level ($p=.06$). Of the tasks with low reliability, Test D [G:L:S] and Test M [N:L:R] showed significant differences in favour of Form B, while the reverse applied to Test B [G:L:C:]. The remaining task, Numerical: Matrix: Generate, showed a significant improvement from the first to the second administration.

Order of Difficulty. It will be recalled that

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TABLE 7

PARALLEL FORMS OF THE PATTERN PROCESSING TEST:
MEANS AND STANDARD DEVIATIONS
(N=30)

	FORM A		FORM B	
	Mean	SD	Mean	SD
G:L:I	7.63	2.24	7.74	2.19
G:L:C	6.11	2.68	5.74	2.99
G:L:R	4.96	2.71	5.44	2.92
G:L:S	7.18	2.52	7.33	2.81
G:L:G	4.85	2.93	4.74	3.12
G:M:I	6.23	1.97	5.38	2.47
G:M:C	6.89	3.15	5.81	2.69
G:M:R	4.78	1.95	4.85	2.29
G:M:S	7.56	3.14	7.28	2.49
G:M:G	5.26	3.35	5.24	3.53
N:L:I	6.33	1.65	5.68	2.52
N:L:C	5.54	2.41	5.92	2.30
N:L:R	4.33	1.82	4.96	2.89
N:L:S	5.08	2.48	5.08	2.54
N:L:G	5.96	2.13	6.40	2.19
N:M:I	4.33	2.66	4.92	3.04
N:M:C	5.29	2.73	5.76	2.83
N:M:R	4.33	2.85	3.88	2.87
N:M:S	6.46	2.52	7.29	2.99
N:M:G	4.17	2.13	4.58	2.20

Maximum score for each task = 10

TABLE 8

PARALLEL FORMS RELIABILITY OF THE PATTERN PROCESSING TEST

			Correlation Coefficient	Probability that r = 0
GEOMETRIC	Linear	Interpolate	.425	.027
		Continue	.303	.124
		Reverse	.637	.000
		Select	.300	.130
		Generate	.551	.002
	Matrix	Interpolate	.449	.021
		Continue	.625	.000
		Reverse	.648	.000
		Select	.845	.000
		Generate	.410	.052
NUMERICAL	Linear	Interpolate	.476	.019
		Continue	.857	.000
		Reverse	.259	.222
		Select	.614	.001
		Generate	.747	.000
	Matrix	Interpolate	.776	.000
		Continue	.678	.000
		Reverse	.554	.005
		Select	.724	.000
		Generate	.104	.630

correct responses to items in the Pattern Processing Test were to be accepted by consensus of three elementary mathematics educators. Each judge answered the geometric tasks in a form where the shapes were uncoloured. Then the colours only were gone through. The judges completed the numerical tasks as set out in the procedures (see Appendix A). Complete consensus on the acceptability of responses was reached. In the construction of the Pattern Processing Test, an attempt was made to increase the difficulty of the items within each task. The geometric items followed the order of the Holland Test Series (Appendix B). According to Klahr and Wallace (1970), the evidence to support their claim that the items are arranged in order of difficulty came from a longitudinal study of five and six year old children who each solved over 300 series completion problems.

The numerical items were drawn from a compendium of items gathered from various intelligence tests which have been normed using extensive populations, according to authors' manuals and the seven Mental Measurement Yearbooks edited by Oscar K. Buros from 1938 to 1972.

In a post-administration consideration of the order of difficulty, the number of correct responses for each item in the interpolate, continue, reverse and select tasks was calculated. These totals can be found in Tables 8, 9, 10 and 11. The tabulations may be looked at in two ways: first, to compare the items within a task; and secondly, to compare the total task scores.

In the geometric linear tasks (see Table 9), the percentage of correct responses generally decreases throughout each task, except for one item, #5, in which shapes alternated singly while the colours alternated in pairs. There is a sudden increase in difficulty in #9 and #10 in the interpolate, continue and reverse tasks where the correct response was not present on the task card and had to be inferred from the information given in the five elements.

In the geometric matrix tasks (see Table 10), the expected gradual decrease in the number of correct responses occurred in the continue and select tasks, but not in the interpolate and reverse tasks. In the interpolate task, the interchange of #2 and #3, and #4 and #5 would rectify the situation. The common feature of #2 and #4 is the inclusion of shape-pairs which may have provided the extra difficulty component.

In the reverse task, #3 and #5 proved too difficult for the subjects. With hindsight it can be seen that #3 was varied too much from the pattern description

TABLE 9

GEOMETRIC LINEAR TASKS:

PERCENTAGE OF CORRECT RESPONSES

(N=100)

Item	A [G:L:I]	B [G:L:C]	C [G:L:R]	D [G:L:S]
#1	92	84	59	82
#2	83	86	68	97
#3	92	86	43	86
#4	90	78	87	95
#5	60	50	30	51
#6	80	67	34	79
#7	62	76	32	78
#8	61	66	27	71
#9	22	12	6	73
#10	24	13	5	35
Total	666	618	391	747

TABLE 10

GEOMETRIC MATRIX TASKS:
PERCENTAGE OF CORRECT RESPONSES
(N=100)

Item	F [G:M:I]	G [G:M:C]	H [G:M:R]	I [G:M:S]
#1	S 92 C 87	V 94 H 87	V 71 H 77	V 83 H 81
#2	S 53 C 47	V 78 H 69	V 75 H 80	V 79 H 78
#3	S 81 C 45	V 73 H 58	V 0 H 0	V 80 H 64
#4	S 38 C 16	V 54 H 43	V 42 H 45	V 67 H 61
#5	S 51 C 16	V 37 H 30	V 0 H 1	V 11 H 52
Total	526	623	391	656

S - Shape
C - Colour

V - Vertical
H - Horizontal

upon which the geometric items were based. #5 showed a substantial increase in the number of children who did not complete it. There were 70 responses in the uncompleted category between the two aspects of #4 and 106 for #5. The only differences between #4 and #5, which have identical pattern descriptions, are the shapes and the location of the colours. It seems, therefore, that the content of #5 does not account for the low performance.

In the select task, #5 proved too difficult. Perhaps because of its difficulty, the children did not engage themselves with the problem.

In the numerical linear tasks (see Table 11), the percentage of correct responses generally decreased throughout each task, except for #8 which had more correct responses than #7 and sometimes #6. In each task, the pattern type for #8 was multiples of x, backwards, non-zero, while #7 involved alternating or increasing addends. Alternating multiples was the common feature of #6 in the interpolate task and #9 in the continue and reverse tasks all of which produced low performance.

When the correct responses in the numerical matrix tasks (see Table 12) are combined for each item, the number consistently decreased throughout each task. The horizontal

TABLE 11

NUMERICAL LINEAR TASKS:

PERCENTAGE OF CORRECT RESPONSES

(N=100)

Item	K [N:L:I]	L [N:L:C]	M [N:L:R]	N [N:L:S]
#1	97	99	80	86
#2	97	76	89	63
#3	79	71	50	18
#4	63	53	46	43
#5	58	45	54	75
#6	13	44	41	41
#7	30	22	22	37
#8	43	45	35	32
#9	14	3	1	36
#10	23	9	8	4
Total	517	467	426	435

TABLE 12

NUMERICAL MATRIX TASKS:

PERCENTAGE OF CORRECT RESPONSES

(N=100)

Item	P [N:M:I]	Q [N:M:C]	R [N:M:R]	S [N:M:S]
#1	C 78 P 70	V 88 H 88	V 71 H 50	V 95 H 79
#2	C 41 P 25	V 59 H 49	V 55 H 51	V 80 H 68
#3	C 31 P 30	V 57 H 36	V 47 H 42	V 59 H 54
#4	C 17 P 11	V 39 H 19	V 15 H 17	V 57 H 37
#5	C 10 P 9	V 44 H 4	V 11 H 18	V 4 H 21
Total	322	483	377	554

C - Central V - Vertical
P - Peripheral H - Horizontal

score for #5 of the continue task (4 correct responses) is unexpectedly low as its pattern description which includes backwards non-zero multiples is similar to the relatively easy eighth item in the numerical linear tasks.

As well as an order of difficulty within tasks, the data in Tables 9 to 12 allow consideration of between task difficulty. The total number of correct responses in each task is summarized in Table 13. Generally a linear task has about the same or more correct responses than its corresponding matrix task, the main exception being in the numerical select tasks. On the other hand, the geometric form of a task has consistently more correct responses than its numerical form, except for the geometric reverse tasks.

Although some items of the Pattern Processing Test obviously are inappropriately placed or designed, the number of correct responses indicates an increase in difficulty within tasks as well as between the two forms and between the two arrangements.

Piagetian Tasks

The two conservation tasks and the two classification tasks were taken directly from Piaget's studies which underpin a recognized theoretical framework.

TABLE 13

PATTERN PROCESSING TEST:

CORRECT RESPONSES FOR EACH TASK

(N=100)

	Geometric		Numerical	
	Linear	Matrix	Linear	Matrix
Interpolate	666	526	517	322
Continue	618	623	467	483
Reverse	391	391	426	377
Select	747	604	435	554

As well, these situations have been replicated in numerous research projects. Although children's performances at certain ages vary from study to study, the investigators report similar responses to the particular tasks. On these grounds, the six Piagetian tasks have been included in the present study.

Standardized Tests

Mathematics achievement. Mathematics achievement was measured by the two mathematics subtests of the *Canadian Test of Basic Skills*: Mathematical Concepts and Problem Solving. According to Birch (Buros, 1972), the test battery has such a long line of respected antecedents that its status need never be in doubt. The reviewer praised the technical sophistication of test design and the production of norms which are provided for each subtest as well as for the whole test. Standardization was established on a group of over 30,000 children drawn from a stratified random sample of over 200 schools throughout Canada. Although it makes very few concessions to the newer mathematics syllabuses, Birch maintained that "for the present, this is probably as useful an instrument as exists (p.15)." Reliability coefficients for the two subtests, Mathematical Concepts and Problem Solving, are .82 and .83 respectively.

Intelligence. Intelligence was measured by the Verbal and Nonverbal Batteries, Level 3 Form A of the Canadian Lorge-Thorndike Intelligence Test, which meet generally accepted standards for test construction and standardization procedures, according to Tittle (Buros, 1972). Wright, Thorndike and Hagen (1972) cited an Edmonton study in which a reliability coefficient of .939 for the verbal and .927 for the nonverbal batteries were obtained for Grade three subjects.

Other tests

Number knowledge. The adequacy of this simple test to measure a child's facility with number was based on teacher opinion. The scores of the pilot study subjects were discussed with their two mathematics teachers who considered that the scores reflected the pupils' level of number facility and discriminated between high and low achievers.

Colour discrimination. This simple test of colour discrimination is used in the Reading and Language Centre of the University of Alberta (Dr. W. Fagan, Director) and is regarded as a satisfactory means of determining whether a child has gross problems in colour discrimination. In the present study, there were no children who indicated deficiency in this area.

SUMMARY

In Chapter III, three main aspects of this study have been presented. The first section contained an account of the design and administration of the instruments, followed by a discussion of the research procedures in the next section. The chapter concluded with an evaluation of the instruments.

CHAPTER IV

RESULTS OF THE INVESTIGATION

This chapter is devoted to the results of the investigation. Under each of the four purposes, details of the results of the particular instrumentation are followed by a discussion of the relevant hypothesis. The analyses of the data were carried out on the IBM 360/67 computer using Fortran programs devised by the Division of Educational Research of the University of Alberta.

Overview of the Test Battery

It will be recalled that 100 Grade three subjects were administered a battery of tests in order to assess

pattern processing abilities
conservation of number and area
classification: additive composition
multiplicative composition
mathematics achievement
intelligence
number knowledge
colour discrimination

and the various relationships within and among these variables.

The means and standard deviations of the performance of the sample on the Pattern Processing Test are presented in Table 14; on the Piagetian conservation and classification tasks in Table 15; and on the tests measuring mathematics achievement, intelligence and number knowledge as well as age in Table 16.

I. PATTERN PROCESSING ABILITIES OF GRADE THREE PUPILS

Results of the Pattern Processing Test

The Pattern Processing Test which was used to measure pattern processing abilities was described in Chapter III with the complete test set out in Appendix A. The children were required to interpolate, continue, reverse, select and generate patterns in two forms, geometrical and numerical, and two arrangements, linear and matrix.

The means and standard deviations for the 100 subjects on the hierarchy of 20 tasks were set out in Table 14, while the percentages of correct responses for each item were reported in Tables 9 to 12.

TABLE 14

PATTERN PROCESSING TEST: MEAN SCORES FOR EACH TASK

(N=100)

			Mean	SD
GEOMETRIC	Linear	Interpolate	6.66	2.13
		Continue	6.18	2.53
		Reverse	3.91	2.77
		Select	7.47	2.06
		Generate	4.21	2.69
	Matrix	Interpolate	5.26	2.06
		Continue	6.23	2.83
		Reverse	3.91	2.00
		Select	6.56	2.83
		Generate	3.88	3.37
NUMERICAL	Linear	Interpolate	5.17	1.84
		Continue	4.67	1.93
		Reverse	4.26	2.18
		Select	4.36	2.11
		Generate	4.51	2.04
	Matrix	Interpolate	3.22	2.38
		Continue	4.83	2.38
		Reverse	3.77	2.56
		Select	5.56	2.27
		Generate	4.34	2.31

TABLE 15

PIAGETIAN TASKS:

MEANS AND STANDARD DEVIATIONS

(N=100)

	Mean	SD
Conservation of Number		
Linear Correspondence	2.71	.67
Circular Correspondence	2.84	.51
Conservation of Area		
Barns Task	2.54	.84
Transformed Triangles Task	2.47	.85
Classification		
Additive Composition	2.64	.54
Multiplicative Composition	2.11	.87

TABLE 16

MEAN MATHEMATICS ACHIEVEMENT, INTELLIGENCE AND NUMBER KNOWLEDGE SCORES AND AGE

(N=100)

	Mean	SD
Mathematics Achievement:		
Mathematical Concepts	37.93	8.49
Problem Solving	38.36	9.23
Total	37.93	8.30
Intelligence:		
Verbal	115.22	15.42
Nonverbal	113.44	16.05
Total	114.02	14.44
Number Knowledge	8.41	2.18
Age (months)	104.47	5.54

The children's performance on items with particular characteristics will now be discussed. Some individual items have already been mentioned in Chapter III in the section on order of difficulty, while Chapter V contains a categorization of error strategies employed by the children in the four tasks with preset correct answers (interpolate, continue, reverse and select).

Geometric: Linear. In the geometric linear tasks, items where the shape and colour go together, for example,

G:L:R #4   RS RS GC GC RS RS

had correct responses of 80 per cent or higher. Items in which the rhythm of the shape sequence differed from that of the colour sequence, for instance,

G:L:C #5   GD YT YD GT GD YT

the percentage of correct responses did not exceed 60. In items #9 and #10, the problem introduced by differing rhythms for the shape and colour sequences was increased by the absence of the correct answer from the set of elements in the stimuli as, for example, in

G:L:I #10   RT * BT RS YT BS

The maximum percentage of correct answers on such items was 24.

The percentage of correct responses for all items in the four geometric linear tasks with preset correct answers was shown in Table 9.

In Test E, the children generated four sequences which were scored for pattern in shape and colour. Of the 400 possible sequences, 131 demonstrated pattern in both shape and colour; 23 displayed pattern in shape but not colour; and 67 in colour and not shape. In 42 sequences same shape same colour was featured in the patterns produced as, for example, in

YC BT YC BT YC BT YC

There were nine sequences in which pattern in both shape and colour was impaired by one element as, for instance, in

GS BS GC BC BS RC

The total number of sequences which scored either one or two points was 272. The categorization of scoring sequences for Test E is set out in Table 17.

A selection of the 11 sequences which gained an extra point for complexity is represented below.

RD YW BD RW YD BW RD
RD YW RC YD RW YC RD
YC BW YC GC BW GC YC

Nine children produced four random lines each, that is, 36 sequences, with an additional 21 coming from the rest of the sample. Randomness was restricted to colour in

TABLE 17

TEST E [GEOMETRIC:LINEAR:GENERATE]: CATEGORIZATION OF SCORING SEQUENCES

Pattern Category	Number of Sequences
Two Points	
Both shape and colour	131
One Point	
Colour only	67
Shape only	23
Same shape - same colour	42
Impairment by one element	9
Total	272

seven sequences and to shape on one occasion only. One shape only appeared in 55 sequences and one colour only in 25 sequences.

Geometric Matrix. In the geometric matrix tasks, the scoring procedure for Test F [G:M:I] differentiated between shape and colour. In all five items there were more correct responses on shape than on colour, the differences being 5, 6, 36, 22 and 35. Item #5 is the only one to use three shapes rather than two; however, in items #3, #4 and #5 the four colours cycle. Altogether the maximum percentage of correct responses for the ten items in the geometric matrix tasks where the colours could be said to cycle was 67, and this occurred in G:M:S #4.

Test G [G:M:C] showed a regular increase in the number of errors. Even though Item #3 with 73 correct responses vertically and 58 horizontally used four colours, they did not incorporate cycling and did not seem to present the difficulty found when the colours were cycling.

The low performance of the children on Test H [G:M:R] has been noted already. Although Items #3 and #5, which had no correct responses, are misplaced within the Pattern Processing Test, they do provide more advanced pattern examples suitable for students beyond the scope of the present study. Similarly, #5 of Test I [G:M:S], in proving too difficult for the Grade three subjects, contains useful complexity within the four by four matrix for more mature students.

In Test J, the children generated two matrices which were scored for pattern in the horizontal and vertical directions. Of the 200 possible matrices, 60 displayed pattern in both directions with 39 of them being checkerboard in design. Forty-seven matrices of a random nature were produced, with another 14 nearly random. Fourteen children each made two random matrices.

The non-random matrices excluding the checkerboard patterns were categorized according to the three transformations of reflection, transformation and rotation. The 100 matrices were made up of 13 reflectional, 63 transformational and 24 rotational patterns. The 22 successfully completed matrices excluding those of the checkerboard type, were categorized as 2 reflectional, 6 transformational and 14 rotational. The categorization of the scoring matrices is presented in Table 18.

Seventeen matrices earned two points for complexity, the criterion being the inclusion of three or more colours and shapes. An example of this level of

TABLE 18

TEST J [GEOMETRIC: MATRIX: GENERATE]:

CATEGORIZATION OF SCORING MATRICES

(N=100)

	Pattern in Both Directions	Other Non-random Matrices
Checkerboard	38	1
Reflection	2	11
Translation	6	57
Rotation	14	10
Total	60	79

complexity is shown below.

GD	YC	BW	RS	GD
YC	BW	RS	GD	YC
BW	RS	GD	YC	BW
RS	GD	YC	BW	RS
GD	YC	BW	RS	GD

Five matrices earned one point for complexity. In this case the criterion was the inclusion of three or more colours *or* shapes. Such a matrix is represented below.

YD	GC	YD	GC	YD
BS	GC	BS	GC	BS
YD	GC	YD	GC	YD
BS	GC	BS	GC	BS
YD	GC	YD	GC	YD

Numerical: Linear. Within the numerical linear tasks four items displayed some visual pattern above their numerical sequencing, as in

N:L:C #1 20 25 30 35 40 45 * *

and

67 77
N:L:S #1 11 22 33 44 55 66
65 88

The percentage of correct scores for this type of item ranged from 79 to 99.

Where two addends were alternating, as in

N:I:I #4 5 6 * 9 11 12

and

N:L:R #4 * * 7 8 10 11 13 14

the percentage of correct responses ranged from 63 to 37.

Three items which incorporated increasing addends, as in

N:L:C #7 2 3 5 8 12 17 * *

had 30, 22 and 36 correct responses, whereas the two items with decreasing addends, as in

N:L:I #9 26 20 15 * 8 6

had 14 and 22.

An item with alternating multiples seemed to present a figure-ground situation to the children. The children seemed to experience great difficulty when the alternates were close together, for example, with the multiples of 4 and 6 in

N:L:I #6 4 * 8 6 12 9 (13 correct responses)

and

N:L:C #9 3 4 6 8 9 12 (3 correct responses)

By comparison, less difficulty was experienced when the alternatives were grossly different, for example, the multiples of 10 and 2 in

N:L:R #6 * * 80 12 70 14 60 16 (41 correct responses)

The difficulty became magnified, however, when the sequence was completely reversed as in

N:L:R #9 * * 18 12 12 8 6 4 (1 correct response)

Four items incorporated doubling. The number of correct responses ranged from 23 for

N:L:I #10 1 2 * 8 16 32

to 4 for

12 22
N:L:S #10 1 2 5 10 11
15 20

In Test Q, the children generated five linear sequences each of which could score up to two points depending on its complexity. Of the 500 possible sequences, 80 earned two points, while 281 gained one point, giving a total of 361 scoring sequences. There were five children who did not score at all. Altogether there were 48 sequences which seemed to be without any order. Some, for instance,

9 62 93 4 92 6006

seemed to be quite random in nature. A further 31 sequences did not score as they repeated a relationship which had already gained one point in another sequence.

The categorization of the 361 scoring sequences according to type is set out in Table 19.

Of the sequences gaining two points, there were 29 of multiples not originating at zero, for instance,

3 9 15 21 27

140 170 200 230 260 290

with the constant addend of 4 being the most frequent (8 occurrences).

Alternating sequences amounted to 20 of which 12 involved the decade numbers and another 5 counting by 1s. The other three were

9 10 7 8 5 6 3 4 2 1

6 4 10 8 14 12 18 16

9 6 12 9 15 12

The six sequences with alternating addends

TABLE 19

TEST C [NUMERICAL:LINEAR:GENERATE]:

CATEGORIZATION OF SCORING SEQUENCES

Pattern Category	Number of Sequences
Two Points	
Multiples not originating at 0	29
Alternating sequences	20
Alternating addends	6
A sequence alternating with a constant	6
Triads	7
Doubling	5
Others (More complex)	7
Total	80
One Point	
Multiples - 2 to 10	175
Multiples - 11 to 100	25
Multiples - others	5
Multiples backwards	15
Forwards by 1s	28
Backwards by 1s	9
Odd numbers	4
Visual patterns	3
Incomplete complex patterns	17
Total	281
Grand total:	361

included three of the 1,2 type. The other three were

32 52 62 82 92

8 9 15 16 22

2 3 7 8 12 13

In the six examples where one sequence alternated with a constant, there were three involving counting by 1s. One of these was

78 76 77 78 78

The other three incorporated the multiples of 2, 5, and 10 respectively, the latter being

4 8 14 8

The seven sequences made up of triads as, for example, in

10 11 12 20 21 22 30 31 32

and

1 7 8 2 9 10 3 11 12

all involved counting by 1s.

Four of the five doubling sequences were of the type originating at 1, the other originating at 5.

There were seven sequences with pattern descriptions more complex than any included in the tasks with preset responses. Two sequences featured increasing addends where the differences also formed a pattern.

2 3 5 7 11 15 [1,2,2,4,4]

1 3 5 8 11 15 19 [2,2,3,3,4,4]

Alternating sequences in triples appeared in

5 10 15 16 17 18 23 28 33

while

10 20 30 31 32 33 43 53 63

is an example of a similar kind where the child displayed the complex relationship but with faulty execution, a distinction pointed out by both Bartlett (1958) and Donaldson (1963).

The most complex pattern, which was produced by a girl aged 8.9 years and with an I.Q. of 138, involved alternating decreasing differences, a type not included in the Pattern Processing Test.

56 50 45 39 34

It may be noted that the proportion of sequences earning two points per subject for each of the 11 classes ranged from 1.54 to .14, the intermediate proportions being 1.44, 1.33, .90, .66, .60, .55, .43, .33 and .33.

Of the 281 sequences which scored one point, 205 were runs of multiples, the most frequent being the 2s with 50 instances. Another 15 featured multiples going backwards with the 2s and 10s each appearing in six sequences. The distribution of the sequences involving multiples is set out in Table 20.

Counting by 1s forwards accounted for 28 sequences, and backwards for 9. The odd numbers appeared in four sequences and visual patterns, for example,

100 100 10 100 100 10

in three.

The sequences categorized as incomplete complex patterns generally indicated that the child had a particular pattern in mind but did not supply sufficient information to establish the relationship clearly. An example like

6 4 7 4 14

illustrates the lack of certainty. Two more elements would have determined whether the child intended his sequence to have alternating addends about a constant, a quite complex pattern description. Altogether there were 17 sequences assigned to this category.

Numerical: Matrix. In the numerical matrix tasks, the scoring procedure for Test P [N:M:I] differentiated between the correct responses which included the central number of the matrix and those correct responses involving only numbers on the periphery of the matrix. Of the

TABLE 20

TEST O [NUMERICAL: LINEAR: GENERATE]:

DISTRIBUTION OF SCORES INVOLVING MULTIPLES

(N=100)

Multiples of	Forwards	Backwards
2	50	6
3	28	0
4	16	2
5	25	0
6	14	0
7	4	0
8	2	0
9	4	1
10	32	6
11	4	0
12	1	0
20	4	0
25	2	0
40	2	0
50	4	0
100	8	0
101	2	0
111	1	0
150	1	0
1000	1	0
Total	205	15

Grand Total 220

possible 500 matrices, 119 were correct in all four numbers and so earned two points each. Of the 84 matrices which scored one point each, 27 had the correct central number, in other words, a number essential to both the horizontal and vertical patterns. The remaining 57 matrices each contained two or three correct numbers, but not including the central number. Table 21 shows the categorization of the correct responses for Test P.

In each of the continue, reverse and select tasks, there was a possible total of 500 correct matrices with patterns in the horizontal and vertical directions. A response was categorized as incomplete if there were not four answers for each line or if the line was not attempted. This category does not include complete but incorrect lines, a response type discussed in Chapter V.

Because of the similar arrangement and scoring procedure for these three tasks, a distinction between the children's performances in the horizontal and vertical directions can be made as well as a comparison between tasks.

Partly successful matrices scored for the pattern in either the horizontal or in the vertical direction. In each of the three tasks the horizontal direction had more

TABLE 21

TEST P [NUMERICAL: MATRIX: INTERPOLATE]:

CATEGORIZATION OF CORRECT RESPONSES

(N=100)

Item	Completely Correct (2 points)	Partially Correct (1 Point)	
		With Central Number	Peripheral Numbers Only
#1	65	5	13
#2	17	8	24
#3	24	6	7
#4	7	4	10
#5	6	4	3
Total	119	27	57

correct responses than its vertical counterpart. Furthermore, more patterns in the vertical direction were not attempted or finished except in the reverse task. The difficulty experienced by the subjects in the reverse task is highlighted by the number of items not completed. Of the possible 500 matrices, 122 were either incomplete or not attempted at all.

As was noted in the earlier discussion on the order of difficulty, the children tended to be more successful on the select tasks than on the other three tasks with preset responses.

The categorization of the scoring and incomplete matrices for the continue, reverse and select tasks is shown in Table 22.

In Test T, the children generated two matrices which were scored for pattern in the horizontal and vertical directions. Of the 200 possible matrices, 109 successfully interlocked linear sequences, with a further 32 partially successful. In 53 of the matrices, the patterning went in one direction only with no attempt by the subjects to consider the other direction. Of these, 12 contained four independent linear sequences, with the other 41 matrices sharing 96 linear sequences. Though four children did not

TABLE 22

TEST Q [N:M:C], TEST R [N:M:R], AND TEST S [N:M:S]: CATEGORIZATION OF SCORING AND INCOMPLETE MATRICES

(N=100)

		Whole	Partically Correct	
		Matrix	Horizontal	Vertical
Test Q [N:M:C]	Scoring	165	122	31
	Incomplete	45	8	37
Test R [N:M:R]	Scoring	129	72	47
	Incomplete	93	32	16
Test S [N:M:S]	Scoring	202	94	56
	Incomplete	33	36	27

attempt the second matrix, there was no attempted matrix which did not contain at least one correct linear sequence.

Of the 20 matrices which scored full points for containing interlocking sequences, 11 combined addends of 2 one way with addends of 10 the other, though the initial number in the top left hand cell differed in ten of them. Another five of these matrices had addends of 10 one way with addends of 20, 5(twice), and 3(twice) respectively the other way. Of the remaining three matrices, one combined addends of 2 and 6 starting at 105. The other two combined forward and reverse sequences of 2 starting at 20, and 10 starting at 65. The latter matrix is represented below.

65	75	85	95
55	65	75	85
45	55	65	75
35	45	55	65

There were 63 matrices which repeated the same type of relationship in both directions, with a further 10 partially successful. Twenty-eight of the successful matrices were based on addends of 2 and ten based on addends of 10, with a further two reversing the 10s patterns.

Of the remaining matrices, seven featured addends of 3, six addends of 5, four addends of 1 and one each of 4s and odd numbers.

17	22	27	32
22	27	32	37
27	32	37	42
32	37	42	47

Two of the remaining four matrices with the same type of relationship going both ways consisted of multiples as set out below,

1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

while two matrices produced by one child were based on doubling. One of his matrices is presented below.

22	44	88	168
44	88	168	336
88	168	336	672
168	336	672	1346

There were 23 matrices involving addends based on four consecutive numbers. Addends of 10 or 2 with various starting points were used in 13 matrices. The other addends were of 3 and of 4, with three matrices featuring backwards counting by 1s. A matrix of this type is presented below.

43	53	63	73
44	54	64	74
45	55	65	75
46	56	66	76

The one matrix involving multiples based on four consecutive numbers is set out below.

5	10	15	20
6	12	18	24
7	14	21	28
8	16	24	32

Two matrices belonged to the fourth category of successful matrices containing 16 consecutive numbers.

In addition to the partially successful matrices which could be categorized according to one of the four types discussed above, there were 15 matrices which showed some evident but disorderly attempt by the subjects to produce interlocking linear sequences.

The categorization of the matrices with at least some evidence of pattern both ways is presented in Table 23.

Summary. In this section, the children's performance on the tests used to explore the hierarchy of 20 tasks has been condensed and discussed. The four generate tasks were dealt with in depth while the error strategies connected with the tasks having preset correct responses will be categorized in Chapter V.

Correlations among the Types of Tasks

The hierarchy of pattern processing tasks can be divided according to process, form and arrangement in order to look for possible relationships within the matrix of tasks.

When the four interpolate tasks are considered, moderate correlations of .44 and .46 ($p<.01$) are seen to

TABLE 23

TEST T [GEOMETRIC:MATRIX:GENERATE]:

CATEGORIZATION OF SCORING MATRICES

(N=100)

	Successful	Partially Successful
Both Directions		
Different patterns	20	4
Doubling	0	1
Same pattern both ways	63	5
Based on 1s	24	3
Involving multiples	2	3
Involving doubling	2	2
Multiples based on 1s	1	0
Consecutive numbers	2	0
Disorderly attempts at two directions	-	15
One Direction	12	41
Total	123	73

Grand Total 196

exist between Test K [N:L:I] and Tests A [G:L:I] and P [N:M:I] respectively and a low correlation of .22 (p<.05) between Test K [N:L:I] and Test F [G:M:I]. Test F also has a low correlation of .23 (p<.05) with Test A [G:L:I]. Test P's numerical matrix task seems isolated from either of the geometric tasks.

When the four continue tasks are considered, a moderate correlation of .48 (p<.01) appears between the two numerical tasks (N:L:C and N:M:C). Low correlations of .20 (p<.05) exist between the three pairs involving the two geometric tasks and the numerical linear. A similar low correlation (r=.21, p<.05) appears between the geometric linear and the numerical matrix tasks. The correlation between the two matrix tasks (G:M:C and N:M:C) does not reach significance (p≥.05).

In the select group, the two forms and arrangements intercorrelate at a moderate level of significance (p<.01) with coefficients ranging from .26 for the two geometric tasks to .51 for the two numerical tasks. The two correlations among the reverse tasks are both moderate. The two linear tasks correlate at .26 (p<.01) with a .47 coefficient (p<.01) for the two numerical tasks.

Among the generate tasks, the two geometric (G:L:G and G:M:G) and the two numerical tasks (N:L:G and N:M:G) have correlations of .38 and .29 (p<.01) respectively, with a .20 (p<.05) correlation for the two linear tasks and .30 (p<.01) for the two matrix tasks. Although the Geometric: Matrix task correlates with the Numerical: Linear task at .39 (p<.01), the parallel pairing of Geometric: Linear and Numerical: Matrix does not reach a significant level of relationship.

The correlations between the 20 tasks grouped according to process are set out in Table 24.

As well as the division of the tasks according to process, arrangement as linear or matrix can form a basis for further comparison. Two geometric tasks, G:L:S and G:M:G, relate significantly (p<.01) to each of the numerical tasks of the same arrangement. As well the Geometric: Select task correlates significantly with all the numerical tasks, except the interpolate, in the matrix arrangement. The task relating least is the reverse task in each arrangement with .26 (p<.01) between the two linear tasks and .21 (p<.05) between geometric reverse and numerical select in the matrix arrangement.

The correlations between the 20 tasks grouped according to arrangement are set out in Table 25.

TABLE 24

CORRELATIONS BETWEEN THE TASKS GROUPED ACCORDING TO PROCESS

(N=100)

Interpolate

	G:L	G:M	N:L	N:M
G:L		.23*	.44**	.17
G:M			.22*	.05
N:L				.46**
N:M				

Continue

	G:L	G:M	N:L	N:M
G:L		.20*	.20*	.21*
G:M			.20*	.19
N:L				.48**
N:M				

Reverse

	G:L	G:M	N:L	N:M
G:L		.07	.26**	-.03
G:M			.16	.08
N:L				.47**
N:M				

Select

	G:L	G:M	N:L	N:M
G:L		.26**	.37**	.28**
G:M			.40**	.41**
N:L				.51**
N:M				

Generate

	G:L	G:M	N:L	N:M
G:L		.38**	.20*	.17
G:M			.39**	.30**
N:L				.29**
N:M				

**Significant at the .01 level of probability
*Significant at the .05 level of probability

TABLE 25

CORRELATIONS BETWEEN THE TASKS GROUPED ACCORDING TO ARRANGEMENT

(N=100)

LINEAR

	G:L:I	G:L:C	G:L:R	G:L:S	G:L:G
N:L:I	.44**	.23*	.12	.31**	.23*
N:L:C	.29**	.20*	.17	.46**	.22*
N:L:R	.18	.09	.04	.27**	.06
N:L:S	.18	.07	.26**	.37**	.21*
N:L:G	.19	.09	.09	.38**	.20*

MATRIX

	G:M:I	G:M:C	G:M:R	G:M:S	G:M:G
N:M:I	.05	.29**	.15	.19	.35**
N:M:C	.27**	.19	.16	.38**	.29**
N:M:R	.19	.20*	.08	.38**	.32**
N:M:S	.19	.31*	.21*	.41**	.38**
N:M:G	.03	.18	.08	.24*	.30**

**Significant at the .01 level of probability
*Significant at the .05 level of probability

When the 20 tasks are grouped according to form, that is geometric and numerical, moderate correlations (p<.01) between all four tasks with preset correct answers appear in the numerical form. As well, the linear generate task has correlations of .28 (p<.01), .20 (p<.05) and .29 (p<.01) with the matrix interpolate, reverse and generate tasks respectively. The linear select select task correlates at the .25 level (p<.01) with the matrix generate task.

Among the geometric tasks, the matrix generate tasks correlate (r=.20 to .38) with all the linear tasks except the reverse task, which correlates at the .27 level (p<.01) with the matrix interpolate task. The matrix select task correlates with the linear continue (r=.33, p<.01) and select (r=.26, p<.01) tasks. The remaining significant correlations (r=.20 to .23, p<.05) occur between the four interpolate and continue tasks in both forms.

The correlations between the 20 tasks grouped according to form are set out in Table 26.

5

TABLE 26

CORRELATIONS BETWEEN THE TASKS GROUPED ACCORDING TO FORM

(N=100)

GEOMETRIC

	G:L:I	G:L:C	G:L:R	G:L:S	G:L:G
G:M:I	.23*	.22*	.27**	.13	.03
G:M:C	.21*	.20*	.14	.16	.13
G:M:R	.14	.09	.07	.17	.05
G:M:S	.19	.33**	.18	.26**	.13
G:M:G	.24*	.20*	.15	.20*	.38**

NUMERICAL

	N:L:I	N:L:C	N:L:R	N:L:S	N:L:G
N:M:I	.46**	.30**	.38**	.43**	.28**
N:M:C	.36**	.48**	.34**	.46**	.16
N:M:R	.44**	.54**	.47**	.52**	.20*
N:M:S	.41**	.45**	.31**	.51**	.12
N:M:G	.15	.19	.16	.25*	.29**

**Significant at the .01 level of probability
*Significant at the .05 level of probability

Factor Analysis

A correlation matrix was calculated for the 20 pattern processing tasks, as well as the six Piagetian tasks. Mathematics achievement and intelligence subtests along with age and number knowledge were included as marker variables.

The correlation matrix for the 33 variables was subjected to Principal Component factor analysis, the factors extracted being rotated by Varimax to approximate orthogonal simple structure. Although there were 11 Eigenvalues equal to or greater than 1.000, the choice of seven factors seemed to provide the most meaningful interpretation.

The minimum loading for inclusion in a principal component was .40, a limit accepted by Kerlinger (1964, p.654). All the variables met this criterion except Additive Composition of Classes which had a loading of .395 on the second factor. Five of the variables, verbal intelligence, the Barns Task, Multiplicative Composition of Classes, Test E [G:L:C] and Test K [N:L:I] had loadings of .40 or more on two factors.

The communalities found among the 7 factors accounted for 58.1 per cent of the total variance existing among the 33 variables.

The factors arising from the factorial treatment of the 33 variables are presented in Table 27.

Factor I, which accounted for 33.2 per cent of the total common variance, contains the eight numerical tasks of the Pattern Processing Test with preset correct answers. Their loadings ranged from .545 to .758. Included also was Geometric: Linear: Select with a loading of .489. Moderate to high loadings appeared in the four marker variables measured by standardized tests. Mathematics achievement is represented by Mathematical Concepts with a loading of .685 and Problem Solving with .736. The loadings on verbal and nonverbal intelligence are .535 and .625, respectively. Number knowledge entered the factor with a loading of .436.

The first factor, identified as Numerical Facility, appears to be schooling based. Current mathematics programs are strongly oriented towards numerical facts rather than spatial relations. The mathematics sections of the Canadian Test of Basic Skills are also inclined to emphasize numerical knowledge. According to Tittle (Buros, 1972), "the Lorge-Thorndike IQ's correlate moderately to fairly highly with school achievement

TABLE 27

FACTORS ARISING FROM THE FACTOR ANALYSIS OF THE PATTERN PROCESSING TEST AND OTHER VARIABLES (N=100)

	COMMUNALITIES	1	2	3	4	5	6	7
1	0.625	0.267	0.153	-0.141	-0.563	-0.106	0.379	-0.195
2	0.646	0.685	0.069	0.257	0.258	0.139	-0.102	-0.099
3	0.624	0.736	0.019	0.236	0.073	0.006	-0.147	-0.012
4	0.393	0.436	-0.026	0.018	0.093	-0.040	0.179	0.399
5	0.684	-0.103	-0.061	0.024	0.007	0.099	0.806	0.098
6	0.721	0.535	0.092	0.320	0.157	0.061	-0.543	0.004
7	0.658	0.625	0.149	0.264	0.105	0.185	-0.330	-0.146
8	0.782	-0.071	0.855	0.008	-0.071	0.013	-0.184	0.080
9	0.717	0.018	0.834	0.007	-0.085	0.011	0.116	0.015
10	0.655	0.039	0.413	-0.043	-0.094	0.260	-0.092	0.629
11	0.577	0.203	0.602	0.119	0.306	0.135	-0.061	0.208
12	0.594	0.180	0.161	0.433	0.157	-0.218	0.231	0.472
13	0.482	0.195	0.395	0.076	0.135	0.316	0.353	-0.201
14	0.534	0.203	0.017	0.614	0.212	0.117	-0.039	0.235
15	0.537	0.133	0.003	0.677	0.046	0.147	-0.155	0.116
16	0.494	0.093	0.062	0.634	-0.074	0.117	0.120	-0.213
17	0.509	0.489	0.207	0.325	0.320	-0.096	0.063	-0.070
18	0.532	0.139	-0.270	0.423	0.491	-0.041	-0.013	0.131
19	0.493	0.176	0.056	0.417	-0.158	0.384	-0.286	-0.176
20	0.615	0.168	-0.058	0.113	0.050	0.724	0.010	0.209
21	0.628	0.102	0.182	0.065	0.152	0.734	0.122	-0.054
22	0.496	0.356	0.063	0.176	0.133	0.560	-0.053	-0.008
23	0.556	0.249	0.114	0.131	0.645	0.206	-0.074	-0.014
24	0.633	0.545	-0.028	0.196	0.123	0.321	0.123	0.405
25	0.669	0.753	0.174	0.192	0.022	0.011	0.039	0.180
26	0.513	0.609	0.028	-0.081	0.014	0.287	0.128	0.191
27	0.580	0.706	0.030	0.062	0.123	0.142	0.182	0.091
28	0.615	0.268	0.107	-0.041	0.675	0.064	0.237	-0.119
29	0.491	0.568	-0.139	0.136	0.323	0.084	0.121	0.069
30	0.613	0.748	0.062	0.089	0.040	0.123	-0.061	-0.146
31	0.677	0.758	-0.061	-0.066	0.143	0.142	-0.218	0.079
32	0.593	0.614	-0.050	0.088	0.193	0.264	-0.244	0.198
33	0.343	0.319	-0.038	-0.089	0.475	0.040	-0.038	0.060
	19.282	6.402	2.458	2.421	2.348	2.265	1.896	1.490

VARIABLES

#1 Sex
#2 Mathematical Concepts
#3 Problem Solving
#4 Number Knowledge
#5 Age
#6 Verbal Intelligence
#7 Nonverbal Intelligence
#8 Number Conservation: Linear Correspondence
#9 Circular Correspondence
#10 Area Conservation: Barns Task
#11 Transformed Triangles Task
#12 Multiplicative Composition of Classes
#13 Additive Composition of Classes
#14 Geometric: Linear: Interpolate
#15 Geometric: Linear: Continue
#16 Geometric: Linear: Reverse
#17 Geometric: Linear: Select
#18 Geometric: Linear: Generate
#19 Geometric: Matrix: Interpolate
#20 Geometric: Matrix: Continue
#21 Geometric: Matrix: Reverse
#22 Geometric: Matrix: Select
#23 Geometric: Matrix: Generate
#24 Numerical: Linear: Interpolate
#25 Numerical: Linear: Continue
#26 Numerical: Linear: Reverse
#27 Numerical: Linear: Select
#28 Numerical: Linear: Generate
#29 Numerical: Matrix: Interpolate
#30 Numerical: Matrix: Continue
#31 Numerical: Matrix: Reverse
#32 Numerical: Matrix: Select
#33 Numerical: Matrix: Generate

(p.686)." The test of number knowledge was strictly factual in nature.

Further support for the description of this factor as schooling based comes from the similarity between the format of the standardized tests and the select tasks of the Pattern Processing Test. All are multiple choice in arrangement, a testing procedure commonly experienced by the children in standardized reading and other tests outside the study. The only geometric task with sufficient loading on this factor was Geometric: Linear: Select, while the remaining select task in the geometric matrix arrangement had a loading of .356 which is approaching the criterion of .40.

Factor II, which appears to be a conservation factor, contained the four Piagetian conservation tasks with loadings ranging from .413 for the Barns Task to .855 for the Linear Correspondence Task. A variable that just failed to meet the criterion was Additive Composition of Classes with a loading of .395. The common feature of this and the conservation tasks is the notion of logical necessity claimed by Piaget as a indication of concrete operational thinking.

The five geometric tasks from the Pattern

Processing Test which made up Factor III with loadings of .417 to .677 required the subjects to supply their own answers even though their correctness was predetermined. Four of the tasks were linear in arrangement, the fifth being Geometric: Matrix: Interpolate. This latter task seems to have no special connection with the linear tasks, except that chronologically it came next in the testing program. The geometric linear task not included (loading of .325) was the multiple choice select task which had a loading of .489 on Factor I.

The other variable which loaded sufficiently on this factor was Multiplicative Composition of Classes. Its connection to the geometric tasks may be the spatial nature of the plastic square and circular blocks used in the testing situation for the classification task.

The third factor may be regarded, therefore, as a Geometric Linear factor because of the predominance of this type of task in the loadings.

Factor IV quite distinctly involves pattern generation as the four generate tasks of the Pattern Processing Test are the only ones whose loadings of .475 to .675 are greater than the criterion. Sex seems to play an important role in this factor. Because girls were labelled

1 as against 2 for the boys for statistical purposes, the negative loading of -0.563 indicates that being a girl was related to the successful generation of patterns as assessed in this study.

Factor V contained three geometric matrix tasks of the Pattern Processing Test, continue, reverse and select, with loadings of .724, .734 and .560 respectively. The loading of .384 on the other geometric matrix task with preset correct answers is approaching the criterion. This factor, therefore, may be identified as geometric matrix patterning.

Factor VI contained two variables neither of which belongs to the Pattern Processing Test. Already loaded on Factor I, verbal intelligence has a loading of .543 on the fifth factor while age has a negative loading of -0.806. Another demographic variable, sex, has a negative loading of -0.379, an indication that being a girl was inclined to influence the likely verbal intelligence score. This factor seems to be specifically verbal intelligence modified by age and sex.

Factor VII is a combination of a conservation task, a classification task and a numerical linear task with loadings of .629, .472 and .405. Number knowledge also enters this factor because of its loading of .399. At first there seems to be no obvious commonality to explain the relationship. However, there is one common feature. Each task involves a part-whole relationship. In the Barns Task the total area of each field is juxtaposed to partially occupied space. Multiplicative Composition required reorganization of the parts of the whole set of plastic blocks. The numerical linear interpolate task incorporates the pattern process of filling a gap in order to complete a sequence whose terminal points are predetermined. However, the other interpolate tasks have low loadings on this factor. The number knowledge test may be regarded as a "filling the gap" exercise because of the physical arrangement of the response sheet. Each sequence required the children to supply a number in a series of ten collinear cells.

Although the two numerical tasks bear only peripherically upon the part-whole relationship, the strength of this concept in the two Piagetian tasks supports the description of Factor VII as the ability to operate with this relationship.

Summary. A factor analysis of the 20 tasks of the Pattern Processing Test along with other variables acting as markers produced seven factors which may be described as

I Numerical Facility

II Conservation

III Geometric Linear Patterning

IV Pattern Generation

V Geometric Matrix Patterning

VI Vertal Intelligence

VII Part-whole Relations

Hypothesis 1. There is no simple structure derivable from Grade three pupils' ability to
- a. interpolate,
- b. continue,
- d. select, and
- e. generate

geometric and numerical patterns when set in linear and matrix arrangement.

When the children's scores on the Pattern Processing Test along with other variables acting as markers were subjected to factor analysis, seven factors appeared. Two of these, II Conservation and VI Verbal Intelligence, were independent of the pattern processing tasks, while the presence of a pattern task in Factor VII, Part-Whole Relations, was minor. Grouping of the Pattern Processing Test by form and arrangement was the focus of Factor III, Geometric Linear Patterning, and Factor V, Geometric Matrix Patterning.

The alignment of the numerical tasks within Factor

I, labelled as Number Facility, emphasized the grouping of the tasks according to form and arrangement rather than to process. A process, however, was the common component of the variables loading on Factor IV, Pattern Generation, which had significant loadings from the four generate tasks and no other variables.

Hypothesis 1 must therefore be rejected on the grounds that all the pattern processing tasks significantly load on factors which group the variables according to form and arrangement, and in one case to process.

II. PATTERN PROCESSING ABILITY AND CONSERVATION

Conservation of number was investigated in two aspects, linear correspondence and circular correspondence. The two conservation of area situations involved the traditional Barns Task and the Transformed Triangles Task. It must be emphasized that the results and relationships derived from the results apply specifically to Grade three children for whom certain Piagetian operations may have been established for some time. Significant relationships of a different kind may have appeared if the sample had consisted of younger children.

Results of the Tests

Of the 100 children in the sample, 83 were categorized as conservers of number on the Linear Correspondence Task, with 90 on the Circular Correspondence Task. There were 82 children who were categorized as conservers on both tasks. The number of nonconservers was 12 for the linear task, 6 for the circular task and 6 for both tasks taken together.

The conservation of area investigation resulted in 77 children being categorized as conservers on the Barns Task, 70 on the Transformed Triangles Task and 61 on both tasks taken together. The number of nonconservers was 23 for the Barns Task, 23 for the Transformed Triangles Task and 4 for both tasks taken together.

When the four conservation tasks were considered as a single test, there were 56 conservers and 4 nonconservers. The conservers, partial conservers and nonconservers for each of the conservation tasks are enumerated in Table 28.

Rationalizations expressed by the conservers of number focussed mainly on the physical correspondence between the two sets of objects prior to any

TABLE 28

DISTRIBUTION OF SCORES ON THE CONSERVATION TASKS

(N=100)

	Conservers	Partial Conservers	Non-Conservers
Conservation of Number:			
Linear Correspondence	83	5	12
Circular Correspondence	90	4	6
Both Tasks	82	12	6
Conservation of Area:			
Barns Task	77	0	23
Transformed Triangles Task	70	7	23
Both Tasks	61	35	4
Conservation of Both Number and Area	56	40	4

transformations.

> They were the same when they were lined up.
>
> When we started there were as many green animals as purple animals.
>
> Same at the beginning.
>
> They're equal. I noticed when you lined them up, you could plainly see that there were as many circles as there were squares.

Some conservers were more concerned with the actual transformations.

> All you're doing is spreading them out and putting them together.
>
> Each of the green animals had a purple animal and you've moved them around and they're the same still.

A few conservers noted that nothing had been removed.

> You didn't take any away and it was the same to start with.
>
> As long as you don't take any away.
>
> When we started we had the same and you never took any away and you never put any in so they are still the same.
>
> Because at the very beginning they were both the same and you never took any away.

The partial conservers lacked firm convictions.

> I don't know.
>
> These [purple animals] are a bigger pile and these [green animals] are not very many, ... Oh, they're probably the same.

The nonconservers focussed on the perceptual differences between the original and transformed arrangements of the two sets of objects in each task.

> [The squares are more] because they are stacked in a pile.
>
> These are in a pile and there are a few purple animals and there are lots of green animals.
>
> It looks more.

The categorization of the children's rationalizations for the Barns Task tends to support the contention that this task is perhaps measuring conservation of number for some children. Of the 77 children classed as conservers, 31 gave spatial rationalizations such as

> Mine are in a row and they're spread out but they're both the same. Same space.
>
> You just put in an equal amount of barns and they are the same size.
>
> They both have as much blocks and cover up as much grass.
>
> Because the barns are the same size and they take up the same space.

On the other hand, 22 children expressed clearly numerical rationalizations.

> Because there are the same number of blocks on each.
>
> Because you added one barn to each side each time.
>
> These are in a line, but these are scattered - but there is the same number

3

of barns.

Eighteen children referred to the "same amount" without distinguishing either a spatial or numerical relationship. A further six children expressed rationalizations which could not be clearly categorized as spatial or numerical.

In the Transformed Triangles Task the rationalizations expressed by the children were clearly spatial and focussed on the initial equivalence of the blocks with the transformations sometimes emphasized, for example,

> Because they're all the same size.
>
> Both are halves and can never be bigger because each half is equal, because each shape is equal.
>
> Because you showed me they were the same - they still are, but you just moved them around.
>
> You can also make them into a square and they're still the same.
>
> Because they all make squares and they were the same size in the beginning.
>
> You just changed it around so it must have as much.

Partial conservers tended to change their minds. One girl claimed the square was larger than the two triangles making up the larger triangle because it was "more squarey than that one." Later she stated that the two transformed triangles had the "same lawns and stuff" as the two rearranged rectangles. A boy who maintained that there was "more in this one [the transformed triangles]" later rationalized the equivalence of the transformed triangles by saying "because that take up the same space."

Another boy who earlier disclaimed equivalence --

> Because it's wider along here, because it looks like more

later stated

> You just have to put this here and they're all the same size, a square.

By contrast, a girl who seemed to recognize the equivalence after a number of rearrangements later argued

> I pictured in my mind that this is a triangle and the middle part is missing and so this [the rearranged rectangle] must be bigger.

Nonconservers tended to focus on the length of the rearranged shapes.

> [The triangles are bigger] because the ends are sticking out and makes more room.
>
> Mine's wider [the square-"larger"] and yours [rearranged rectangles] is skinnier.
>
> This one's shorter and this one's longer [the rearranged triangles-"larger"].
>
> It's [transformed triangles] bigger and wider.
>
> Because it's longer than the square.

> It's [transformed triangles] sorta got more room (pointing to the apex).
>
> Because it's big [transformed triangles] and this [rearranged rectangles] looks so small.
>
> [The square] is shorter than this one [rearranged rectangles-"larger"].

Correlations between the Tasks

All the correlations between the conservation tasks were significant. The two number correspondence tasks correlated at .69 (p<.01). The Linear Correspondence Task had correlations of .40 and .38 (p<.01) with the Barns Task and the Transformed Triangles Task, respectively, while the correlations between the Circular Correspodence Task and the Barns and Transformed Triangles Tasks were .25 (p<.05) and .36 (p<.01), respectively.

The correlation of .33 (p<.01) occurring between the Barns Task and the Transformed Triangles Task is the second lowest among the conservation tasks, and so supports the doubt that the Barns Task is always testing area conservation.

The correlations between the conservation of number and area tasks are shown in Table 29.

TABLE 29

CORRELATIONS BETWEEN THE CONSERVATION TASKS

(N=100)

	Numerical Correspondence		Barns Task	Transformed Triangles Task
	Linear	Circular		
Linear Correspondence		.69**	.40**	.38**
Circular Correspondence			.25*	.36**
Barns Task				.33**
Transformed Triangles Task				

**Significant at the .01 level of probability
*Significant at the .05 level of probability

Hypothesis 2 There are no significant correlations between Grade three pupils' ability to

a. interpolate,
b. continue,
c. reverse,
d. select, and
e. generate

geometric and numerical patterns, and their ability to conserve

a. number, and
b. area.

When the Pearson Product Moment correlations between the Pattern Processing Test and the conservation tasks were calculated, Conservation of Number in either the linear or circular correspondence aspects did not correlate significantly with any of the 20 pattern processing tasks for Grade three pupils. Hypothesis 2 must therefore be accepted with regard to the children's ability to conserve number.

The only significant correlation involving the Barns Task occurred at the .29 level ($p<.01$) with Test K [N:L:I].

Because the Barns Task has connotations of number conservation, the Transformed Triangles Task is regarded as the means of measuring conservation of area within this section of the study.

The Transformed Triangles Task correlated with 13 of the pattern tasks. The .01 level of probability was reached with Gecmetric: Linear: Select, Geometric: Matrix: Reverse and Generate, and Numerical: Linear: Interpolate. The .05 level of probability was reached with Geometric: Linear: Interpolate, Geometric: Matrix: Select, Numerical: Linear: Continue, Reverse and Generate, and the four numerical matrix tasks with preset correct answers. No common relationship seems evident in the particular tasks which contributed to significant correlations.

Hypothesis 2 is rejected for 13 of the pattern tasks with regard to the Grade three children's ability to conserve area, and is accepted for the remaining seven tasks, Geometric: Linear: Continue, Reverse and Generate, Geometric: Matrix: Interpolate and Continue, Numerical: Linear: Interpolate and Numerical: Matrix: Generate.

The correlations between the Pattern Processing Test and the four conservation tasks are shown in Table 30.

III. PATTERN PROCESSING ABILITY AND CLASSIFICATION

Two classification skills were investigated in this study, Additive Composition of Classes by a version of the classic Wooden Beads test and Multiplicative Composition of Classes using circles and squares in two colours.

TABLE 30

CORRELATIONS BETWEEN THE PATTERN PROCESSING TEST AND THE CONSERVATION TASKS

(N=100)

	Numerical Correspondence			
	Linear	Circular	Barns Task	Transformed Triangles Task
G:L:I	-0.03	.05	.12	.22*
G:L:C	.08	.03	.02	.17
G:L:R	.02	.05	.06	.09
G:L:S	.09	.08	.02	.29**
G:L:G	-0.14	-0.19	-0.06	.08
G:L:I	.04	.09	-0.05	.13
G:M:C	.04	-0.01	.22*	.15
G:M:R	.07	.12	.20	.28**
G:M:S	.09	-0.02	.10	.23*
G:M:G	.05	.11	.10	.32**
N:L:I	-0.11	.05	.29**	.26**
N:L:C	.10	.16	.15	.25*
N:L:R	-0.05	.09	.15	.22*
N:L:S	-0.02	.04	.14	.19
N:L:G	-0.01	.02	-0.05	.20*
N:M:I	-0.19	-0.03	.04	.22*
N:M:C	.03	.05	-0.04	.22*
N:M:R	.03	-0.04	.04	.23*
N:M:S	.03	.01	.15	.21*
N:M:G	-0.08	-0.06	.01	.17

**Significant at the .01 level of probability
*Significant at the .05 level of probability

Results of the Test

Of the 100 Grade three children in the sample, 44 were categorized as classifiers on Additive Composition or class inclusion, 23 as partial classifiers and 33 as nonclassifiers. On the multiplicative composition task, there were 67 children categorized as classifiers, 30 as partial classifiers and 3 as nonclassifiers.

When the two classification tasks were taken together, there were 34 children categorized as classifiers, 65 as partial classifiers and 1 as a nonclassifier. The correlation of .10 between the two tasks was not significant.

The classifiers, partial classifiers and nonclassifiers on the two tasks taken separately and together are enumerated in Table 31.

For the Additive Composition of Classes task involving plastic tiles in two colours the entire sample recognized that the two red tiles were indeed plastic. The majority of the children categorized as classifiers succinctly rationalized with the words, "They're all plastic." Other statements elaborate the children's thinking.

TABLE 31

DISTRIBUTION OF SCORES ON THE CLASSIFICATION TASKS

(N=100)

	Classifiers	Partial Classifiers	Non-Classifiers
Classification:			
Additive Composition	44	23	33
Multiplicative Composition	67	30	3
Both tasks	34	65	1

> Because the plastic squares include the red squares too.
>
> If you put all these [the green tiles] away there'd still be two plastic left.
>
> If you put all the greens in one row you wouldn't have the two reds and if you have the plastic row you'd have the two reds too.
>
> Because the two reds count too.
>
> Because they're all plastic and not all green.

All except the children categorized as partial classifiers maintained initially that there were "more green than red" when requested to compare the row of plastic tiles to the row of green tiles. Two children explained that the green row was longer than the plastic row.

Later in the testing situation, the partial classifiers acknowledged that the row of plastic tiles was longer than the row of green tiles without any apparent awareness of their inconsistency.

> The red ones plastic too so if you include the plastic red ones would be two more.
>
> Green and red are all plastic.
>
> Because two plastics are added to these [the green tiles].

One child denied initially that the plastic row was longer than the green row yet stated, "They're all plastic but they're not all green". At the end of the test, he rationalized the dilemma embodied in his decision making:

> Problem is they're all plastic. Most of them are green. Because all of them are plastic.

The children categorized as nonclassifiers consistently claimed that the green row was longer than the plastic row. Their rationalizations indicate their logical confusion.

> Green row longer because only two red ones.
>
> Green ones more even though the red ones are plastic.
>
> Only two reds and there are more greens.
>
> I don't know.
>
> Greens and reds are all plastic but there are more greens than reds.
>
> Reds only have two, the greens have more than that.

In the Multiplicative Composition of Classes task, which involved the grouping and regrouping of coloured plastic blocks, the children categorized as classifiers expressed no doubts about the concurrent criterial properties of the objects as colour and shape.

The children categorized as partial classifiers can be subdivided. Twenty-four children agreed that the plastic pieces could be classified according to colour but not to the subsequent criterion of shape. Four children who denied that the plastic blocks could be classed according to colour subsequently acknowledged the grouping of the blocks according to shape.

Three children claimed that the plastic blocks could not be regrouped at all, according to either colour or shape.

When requested to sort the plastic blocks, one child did not utilize the properties of colour or shape but concentrated on pictorial functions. The themes of marine transport and bombardment were continued throughout the regrouping part of the test.

Correlations between the Classification and Conservation Tasks

When correlations between the two classification tasks and the four conservation tasks were calculated, those involving the Linear Correspondence and Barns Tasks did not reach significance ($p \geq .05$). Multiplicative Composition of Classes correlated with the Transformed Triangles Test at the .28 level ($p < .01$), while Additive Composition of Classes correlated with Circular Correspondence and the Transformed Triangles Test at .29 and .30 (p<.01) respectively.

The correlations between the six Piagetian tasks are shown in Table 32.

Hypothesis 3. There are no significant correlations between Grade three pupils' ability to

- a. interpolate,
- b. continue,
- c. reverse,
- d. select, and
- e. generate

geometric and numerical patterns, and their ability to classify:

- a. additive classes, and
- b. multiplicative classes.

When the Pearson Product Moment correlations between the Pattern Processing Test and the two classification tasks were calculated, there were seven significant correlations (r=.21 to .36) which involved Additive Composition of Classes, with 8 (r=.20 to .32) for Multiplicative Composition. Four of the geometric tasks were common to both lists, Linear: Interpolate and Matrix: Interpolate, Continue and Select. The tasks correlating significantly with Multiplicative Composition only were Geometric: Linear: Continue and Select, and Numerical: Matrix: Interpolate. For Additive Composition, the additional significant correlations occurred with Geometric: Matrix: Reverse and Select, Numerical: Linear: Generate, and

TABLE 32

CORRELATIONS BETWEEN THE CLASSIFICATION AND CONSERVATION TASKS

(N=100)

	Classification	
	Multiplicative Composition	Additive Composition
Conservation:		
Linear Correspondence	.07	.16
Circular Correspondence	.08	.29**
Barns Task	.16	.04
Transformed Triangles Tasks	.28**	.30**

**Significant at the .01 level of probability

Numerical: Matrix: Continue. The tasks with which neither classification skill correlated significantly were the two Geometric:Continue tasks and the two Geometric: Generate tasks, with three numerical matrix tasks, reverse, select and generate.

The correlations between the Pattern Processing Test and the two classification skills are shown in Table 33.

Although the two classification tasks between them correlated significantly with 12 of the pattern processing tasks, Hypothesis 3 can be rejected for specific tasks only with regard to the two classificatory skills taken separately. For more than half the pattern tasks, Additive Composition as well as Multiplicative Composition did not correlate significantly, a situation which leads to the acceptance of the hypothesis in these instances.

IV. PATTERN PROCESSING ABILITY AND OTHER VARIABLES

The other four variables include two of a demographic nature, sex and age, as well as mathematics achievement and intelligence each of which was measured by two subtests.

TABLE 33

CORRELATIONS BETWEEN THE PATTERN PROCESSING TEST AND THE CLASSIFICATION TASKS

(N=100)

	Multiplicative Composition	Additive Composition
G:L:I:	.11	.23*
G:L:C:	.29**	.10
G:L:R:	.22*	.06
G:L:S:	.09	.19
G:L:G:	.35**	-0.09
G:M:I:	.18	.04
G:M:C:	.12	.13
G:M:R:	.05	.32**
G:M:S:	.10	.22*
G:M:G:	.10	.15
N:L:I:	.26**	.23*
N:L:C:	.37**	.21*
N:L:R:	.06	.26**
N:L:S:	.25**	.20*
N:L:G:	.13	.17
N:M:I:	.21*	.13
N:M:C:	.15	.27**
N:M:R:	.11	.16
N:M:S:	.12	.13
N:M:G:	.15	.08

**Significant at the .01 level of probability
*Significant at the .05 level of probability

Results of the Tests

The mean scores and standard deviations for mathematics achievement and intelligence as well as age were shown in Table 15. The sample's mean grade score of 3.7 for mathematics achievement was close to the 3.5 expected in March. The mean intelligence score of 114 indicates that the sample as a whole was slightly above average. In age, the mean of 8.8 years is normal for Grade three children in March.

Comparative Performance on the Pattern Processing Test

The child with the lowest performance on the Pattern Processing Test gained a total of 38 points, with a score of zero on seven tasks. His intelligence rating was 96, with a mathematics achievement grade score of 2.6. One child had 5 zero scores, another 4 and 16 children had 2 zero scores each. The intelligence and mathematics achievement scores of these children were distributed over the full range for the sample. Of the 18 children in the sample with intelligence scores below 100, eight had two or more zero scores on the Pattern Processing Test. Moreover, on the Pattern Processing Test, no child's highest score was less than five.

At the other end of the scale, nine children with two or more scores of 10 and 9 shared 25 scores of 10 and 31 scores of 9. The tens scores are spread over 14 tasks with the interpolate and geometric linear blocks having the highest tallies. With one exception, this group of nine high performers had intelligence scores of 123 or better and were advanced one year at least in mathematics achievement. The one exception had an intelligence rating of 110 and was just below the expected grade level in mathematics achievement. All nine children scored 9 or 10 on the Number Knowledge test.

Just as the children with below average intelligence did not all perform poorly on the Pattern Processing Test, the opposite was also true. Many children with high intelligence and mathematics achievement scores did not perform exceptionally well on the pattern tasks.

Hypothesis 4. There are no significant correlations between Grade three pupils' performance on the Pattern Processing Test and

a. mathematics achievement,
b. intelligence,
c. age, and
d. sex.

The Mathematical Concepts section of the Canadian Test of Basic Skills correlated significantly with every pattern processing task, 16 at the .01 level of probability and the remaining four at the .05 level. Problem Solving correlated with 13 of the pattern processing tasks at the .10 level of probability, and with 4 at the .05 level. Three geometric tasks did not correlate significantly, Linear: Continue and Matrix: Continue and Reverse.

In the light of these results, it seems justified to reject Hypothesis 4 with regard to mathematics achievement.

Verbal Intelligence correlated with 13 of the pattern processing tasks at the .01 level of probability and four at the .05, Geometric: Linear: Continue, Geometric: Matrix: Reverse: and Numerical: Linear: Generate. Nonverbal Intelligence correlated with 19 of the 20 pattern processing tasks, 14 at the .01 level. The only task not reaching significance was Numerical: Matrix: Generate.

As all the pattern processing tasks correlated significantly with at least one aspect of intelligence, Hypothesis 4 is rejected in respect to intelligence in both its verbal and nonverbal forms.

The correlations between the Pattern Processing Test, mathematics achievement and intelligence are shown in Table 34.

TABLE 34

CORRELATIONS BETWEEN THE PATTERN PROCESSING TEST AND MATHEMATICS ACHIEVEMENT AND INTELLIGENCE

(N=100)

	Mathematics Achievement		Intelligence	
	Mathematical Concepts	Problem Solving	Verbal	Nonverbal
G:L:I	.31**	.25*	.35**	.30**
G:L:C	.27**	.31**	.33**	.29**
G:L:R	.24*	.13	.17	.24*
G:L:S	.43**	.45**	.38**	.44**
G:L:G	.34**	.22*	.26**	.23*
G:M:I	.25*	.23*	.34**	.29**
G:M:C	.23*	.23*	.22*	.22*
G:M:R	.30**	.18	.22*	.24*
G:M:S	.38**	.32**	.26**	.40**
G:M:G	.46**	.32**	.28**	.28**
N:L:I	.40**	.39**	.35**	.40**
N:L:C	.52**	.56**	.41**	.54**
N:L:R	.31**	.35**	.29**	.40**
N:L:S	.48**	.39**	.32**	.47**
N:L:G	.30**	.21*	.15	.25*
N:M:I	.51**	.44**	.36**	.37**
N:M:C	.51**	.51**	.43**	.51**
N:M:R	.53**	.53**	.46**	.46**
N:M:S	.50**	.48**	.41**	.45**
N:M:G	.24*	.21*	.22*	.14

**Significant at the .01 level of probability
*Significant at the .05 level of probability

==

Sex correlated significantly with the two geometric generate tasks for which the negative correlations of -0.23 ($p<.05$) and -0.27 ($p<.01$) indicate the influence of being a girl. Similarly, the only pattern processing task with which age correlated significantly was Geometric: Matrix: Interpolate ($r=-0.23$, $p<.05$).

In view of these results, Hypothesis 4 is accepted for both sex and age.

The correlations between the Pattern Processing Test, sex and age are shown in Table 35.

SUMMARY

Chapter IV contained the results of the instrumentation followed by the testing of the hypotheses associated with the four major purposes of the present study. In this chapter the pattern generating tasks were discussed in detail, while the error strategies employed by the children on the pattern tasks with preset correct answers will be categorized in Chapter V. A discussion of the major findings from both Chapters IV and V will be presented in the sixth and final chapter, with implications for curriculum developers and teachers in the classroom,

TABLE 35

CORRELATIONS BETWEEN THE PATTERN PROCESSING TEST, SEX AND AGE

(N=100)

	Sex	Age
G:L:I:	-0.14	-0.06
G:L:C:	-0.16	-0.00
G:L:R:	.05	.01
G:L:S:	-0.05	-0.09
G:L:G:	-0.23*	.07
G:M:I:	-0.05	-0.23*
G:M:C:	-0.06	.04
G:M:R:	.02	.06
G:M:S:	-0.10	.03
G:M:G:	-0.27**	.12
N:L:I:	-0.08	.05
N:L:C:	.07	-0.10
N:L:R:	.06	.05
N:L:S:	.09	.04
N:L:G:	-0.17	.05
N:M:I:	-0.02	.05
N:M:C:	.14	-0.06
N:M:R:	-0.03	-0.12
N:M:S:	-0.14	-0.11
N:M:G:	-0.01	-0.07

**Significant at the .01 level of probability

*Significant at the .05 level of probability

together with recommendations for future research.

CHAPTER V

CATEGORIZATION OF ERROR STRATEGIES EMPLOYED IN THE PATTERN PROCESSING TEST

In Chapter IV the emphasis in the reporting of the results of the Pattern Processing Test was on the performance of the children according to some criterion of success. In Chapter V the focus is on the strategies employed by the children in those items of the tasks with preset correct answers where their performance did not meet the stipulated criterion of success. The error strategies have been categorized to varying depths depending on the amount of information residing in the data. As the responses produced by the children are assumed to be the overt manifestations of mental operations, the error strategies are drawn from the interpretation of indirect evidence.

The various categories of error strategies are briefly described as they arise, even though the highest frequency for a particular type may occur in a task reported later.

With the exception of Test P [Numerical: Matrix:

Interpolate], each of the tasks with preset correct answers had 1000 possible responses over the total sample. In order to categorize the responses to the Numerical: Matrix: Interpolate task in a meaningful way, each of the five items was viewed as a whole so that the maximum number of responses for the purposes of this chapter was 500.

A detailed description of the Pattern Processing Test, including the scoring procedure, is set out in Appendix A.

Geometric: Linear

The geometric linear tasks were contained within a limited closed system having a maximum of four colours, four shapes and nine elements in any one sequence. For each of the four tasks the most frequently employed error strategy seemed specific to the pattern process involved. In the interpolate task there were 54 instances in which the children reproduced one of the elements adjacent to the gap, while in 56 cases of the continue task, part of the given sequence was reproduced as the response. One of the most frequent error strategies among all the tasks of the Pattern Processing Test occurred in the linear reverse task. Two hundred and thirty-seven responses contained the correct elements but in the reverse order even though this aspect was specially discussed before the test was administered. Moreover, no subject consistently employed this strategy in all his responses and so this strategy of reversing the order does not seem to result from the misinterpretation of test instructions. In the select task the most commonly chosen response was a repetition of the final element in the given response, a strategy which occurred 50 times.

A strategy employed frequently in three of the four tasks was the repetition of the element(s) at the left hand end of a given sequence (G:L:I, 42; G:L:C, 50; G:L:S, 72). There were only eight occurrences in the reverse task where a higher frequency may have been expected because of the emphasis on the left hand end.

"Fairness", one of the types of rationalizations noted by Klahr and Wallace (1963) in their study of children's pattern responses, occurred 23 times in the interpolate task of the present study. The need for each element to have a partner seems connected with the child's sense of symmetry as, for example, in

G:L:I #8 BS BS RC RE RC BS BS BC

Loss of hold is the term used by Bartlett (1958) and Donaldson (1963) to describe the strategy whereby a subject's response is correct initially, then appears to change course, as for instance in

G:L:C #9 YD BH GD BD GH GD

Three sequences in the continue task fell into this category.

The total numbers of errors categorized by strategy in the interpolate, continue, reverse and select tasks were 138, 155, 367 and 220, respectively, with 61, 30, 51 and 14 errors remaining uncategorized.

The categorization of the error strategies employed in the geometric linear tasks is set out in Table 36.

Geometric: Matrix

In the four geometric matrix tasks with preset correct answers, the recorded evidence of the children's mental processing of pattern was circumscribed by the constraints inherent in the matrix arrangement of a small closed system of coloured shapes, as well as by the requirement to fill only one row of cells in the three extrapolation tasks (continue, reverse and select). As a result a detailed categorization of the error stategies was not attempted for these tasks.

TABLE 36

GEOMETRIC LINEAR TASKS:

CATEGORIZATION OF ERROR STRATEGIES

(N=100)

Error Category	A [G:L:I]	B [G:L:C]	C [G:L:R]	D [G:L:S]
Right end element(s)				
Repeated	19	25	35	80
Reversed	0	3	13	20
Left end element(s)				
Repeated	42	50	8	72
Reversed	0	9	2	0
Correct elements in reverse order	0	0	237	0
Adjacent element repeated	54	0	0	0
Continued in wrong direction	0	0	17	0
Part of sequence				
Repeated	0	56	47	34
Reversed	0	9	7	14
Loss of hold	0	3	0	0
"Fairness"	23	0	1	0
Total	138	155	367	220
Not in set	12	9	15	0
Indecipherable	0	2	1	0
Uncategorized	61	30	51	14
Grand Total	211	196	444	234

The children seemed divided into two groups, those who could solve the problem easily and those who had great difficulty in doing so at all. In the continue task, for instance, 29 children scored either 9 or 10, while 21 managed scores of 3 or less when the first item worth two points was designed on Burt's (1922) advice that the initial test should act as a shock absorber whose purpose is "to allay the nervousness and engage the interest of the child (p.9)."

The less successful children in the interpolate task tended to concentrate on the shape property of the four missing elements in each matrix before attacking the matter of colour. Altogether there were 351 errors in either shape or colour. It is noticeable, however, that in this task, the first involving the geometric matrix arrangement, the children did at least strive to solve the five matrix problems.

In the three extrapolation tasks the children who were not successful seemed to have only two strategies available to them; either to repeat one of the given lines of elements or to ignore the problem altogether. The number of items not attempted increased across the tasks from 82 in the interpolate task to 191 in the reverse task. In providing choices the select task with 138 matrices not attempted allowed the less successful childen an escape from their dilemma.

The broad categorization of the responses to the geometric matrix tasks is presented in Table 37.

Numerical: Linear

Between them, the two most frequent error strategies employed in the numerical linear tasks accounted for 849 out of a total of 1810 error responses. The first strategy involved the next consecutive counting number either forwards or backwards. In the select task where one of the distractors consisted always of the next consecutive number, there were 293 occurrences of this strategy. In the interpolate task this strategy was applied to the two numbers adjacent to the missing element on 122 occasions.

In the second of the two most frequent strategies the children produced multiples associated with either the end number in the three extrapolation tasks or the numbers adjacent to the missing element in the case of the interpolate task. This strategy sometimes took the form of multiples originating at 0 as in

N:L:C #4 2 3 5 6 8 9 12 18

A variation retained the multiples in the ones digits as in

TABLE 37

GEOMETRIC MATRIX TASKS:

CATEGORIZATION OF RESPONSES

(N=100)

Response Category	F [G:M:I]	G [G:M:C]	H [G:M:R]	I [G:M:S]
Correct	526	623	391	604
Attempted but not finished	41	83	46	10
Not attempted	82	172	191	138
Wrong	351	122	372	248
Total	1000	1000	1000	1000

N:L:C #6 4 5 8 9 12 13 16 19

In the select task where some form of multiple associated with the end number was one of the choices, there were 159 instances of children employing this error strategy, while the interpolate, continue and reverse tasks had 102, 51 and 85 instances, respectively.

Altogether, these two strategies accounted for 51.6 per cent of the interpolate error strategies, 17.3 per cent of the continue, 31.1 per cent of the reverse and 71.6 per cent of the select error responses.

Other error responses in the interpolate task included the difference between the two numbers adjacent to the missing element on 59 occasions, and the next odd (or even) number if the numbers adjacent to the missing element were odd (or even) with 50 instances. A local solution which noted a relationship between the numbers near the missing element but disregarded the whole sequence was produced on 20 items. In general, the children seemed to concentrate on the numbers adjacent to the missing element as there were 383 error responses relating to the adjacent numbers and only 13 and 21 relating to the right and left end numbers, respectively.

The number of error responses in the Numerical: Linear: Interpolate task remaining uncategorized was 17.

The categorization of the error strategies for Test K [N:L:I] is shown in Table 38.

In the continue task, there were 113 responses which indicated that the child was obviously bordering on successful discovery of the relationship involved, yet the execution was not close enough for the response to be considered as merely inaccurate. Examples of this proximal type of response are

N:L:C #5 36 32 28 24 20 16 11 6
N:L:C #8 47 42 37 32 27 22 18 13

A group of 122 responses which could not be categorized according to error strategy showed that the children's thinking was not disorderly, yet there was insufficient evidence to justify describing them as proximal responses. Examples of responses regarded as not disorderly are

N:L:C #2 21 19 17 15 13 11 8 5
N:L:C #8 47 42 37 32 27 22 19 13

There were 68 error responses remaining uncategorized for the continue task.

TABLE 38

TEST K [NUMERICAL: LINEAR: INTERPOLATE]:

CATEGORIES OF ERROR STRATEGIES

(N=100)

Error Category	Frequency
Relative to Either Adjacent Number	
Next number (forwards or backwards)	122
Next odd (if odd) or even (if even) number	50
Associated multiple	102
Next decade number	12
Local solution	20
Difference between the two adjacent numbers	59
Inaccurate	18
Relative to Final Number	13
Relative to Initial Number	21
Uncategorized	17
Total	434

The matter of direction characterizes the pattern processing problem specific to the reverse task. There were 31 responses which contained the correct elements in the wrong order. Of the 54 proximal responses, 9 were in the reverse order while 15 of the 81 responses described as not disorderly were also in the reverse order. Three responses continued a sequence in the wrong direction, an indication that the child was changing from the reverse to the continue task. Nevertheless, out of the 421 error responses, only 58 were the result of reversing direction which, therefore, cannot be regarded as the most significant factor in the children's lack of success in this task.

The number of error responses remaining uncategorized for the reverse task was 101.

The format of the select task restricted the number of error strategies available to the children. Of the 502 error responses, 452 were categorized as belonging to either the next number or the associated multiples types as previously noted. In the third most frequent category (24 occurrences), the children chose the next decade number, as in

N:L:S #9 8 10 13 17 22 28 30

The number of error responses remaining uncategorized in the select task was 18.

The categorization of the error strategies for Test L [N:L:C], Test M [N:L:R] and Test N [N:L:S] is presented in Table 39.

Numerical: Matrix

The structure of Test P [Numerical:Matrix: Interpolate] seemed to provide the children with the opportunity to reveal the state of their mathematical thinking through the error strategies they employed. The task called for the coordination of linear sequences in two directions. There were 70 matrices in which the children filled the four empty cells so that independent linear sequences resulted, as in the following example.

N:L:I #2

5	10	15
7	14	21
9	18	19

On eight occasions the children found that after using three cells to construct uncoordinated linear sequences there was

TABLE 39

TEST L [N:L:C], TEST R [N:M:R] AND TEST S [N:M:S]: CATEGORIZATION OR ERROR STRATEGIES

(N=100)

	L [N:L:G]	M [N:L:R]	N [N:L:S]
End element(s)			
Repeated	1	2	8
Reversed	0	0	0
Next number(s) (forwards or backwards)	37	47	293
Next number(s) in odd or even sequences	41	17	0
Next decade	5	0	24
Associated multiple(s)	51	85	159
Correct elements in wrong order	0	31	0
Continued in wrong direction	0	3	0
Loss of hold	3	0	0
Inaccurate	5	0	7
Proximal response	113	45	0
Reverse order	0	9	0
Total	256	239	491
Not disorderly	122	66	0
Reverse order	0	15	0
Uncategorized	68	101	18
Grand Total	446	421	509

no sequence that could accommodate the final cell.

N:L:I #1

2	4	6
4	8	2
6	12	10

Eighty-five matrices contained uncoordinated linear sequences except for one cell which was filled by a number associated with an adjacent number, as in

N:L:I #4

20	14	14
30	15	12
16	16	10

There were seven matrices in which each empty cell was filled with a number associated with an an adjacent element, as in

N:M:I #3

10	20	30
13	19	25
16	12	36

In 18 matrices the children used the operations of addition or subtraction to relate adjacent numbers. The three by three format may have drawn some children to view the items as number fact puzzles, a type of exercise often found in elementary mathematics texts. Yet there were a further 25 matrices containing linear sequences as well as number bonds. This error strategy involving the operations may be another example of the loss of hold strategy.

The number of matrices remaining uncategorized in the Numerical: Matrix: Interpolate task was 47.

The categorization of the error strategies for Test P [N:M:I] is set out in Table 40.

In the three extrapolation tasks, continue, reverse and select, the loss of hold strategy seems evident in those response lines which were correct except for one

TABLE 40

TEST P [NUMERICAL: MATRIX: INTERPOLATE]: CATEGORIZATION OF ERROR STRATEGIES

(N=100)

Error Category		Frequency
Matrix correct except for one inaccuracy		8
Uncordinated linear sequences		
All cells		70
"Trapped" on final cell		8
Spoilt by a response associated with an adjacent number		85
Each response associated with an adjacent number		7
Operations used to relate adjacent numbers		
All cells		18
Mixed with linear sequences		25
	Total	221
One cell inexplicable		16
Not disorderly		43
Uncategorized		47
	Grand Total	327

number associated with an adjacent element from the given matrix. The number spoiling the response line was usually some form of multiple. There were 38, 30 and 85 occurrences of this strategy in the three tasks, respectively.

Although the frequency with which the response lines formed independent linear sequences was 115, 110 and 20 for the three tasks respectively, there were few instances (13, 0 and 3, respectively) where the loss of hold strategy was observed in one cell of an independent linear sequence. However, there were 64, 12 and 65 error responses, respectively, in which each cell was filled with a number related to the adjacent element of the given matrix, but unrelated to either the linear sequences comprising the matrix or the response line being produced.

There were more examples of inaccuracy in the three extrapolation tasks in the matrix arrangement than in any other group of tasks. One inaccurate number spoilt otherwise correct response lines on 57, 40 and 14 occasions, respectively.

The number of response lines remaining uncategorized were 42, 117 and 51, respectively.

The categorization of the error strategies for

Test Q [N:M:C], Test R [N:M:R] and Test S [N:M:S] is set out in Table 41.

Summary

In this chapter the children's error responses on the Pattern Processing Test have been categorized according to strategy. An exhaustive analysis, however, was not pursued, especially in the geometric matrix tasks where a detailed categorization did not seem meaningful. In the remaining 12 tasks the number of possible responses was 11,500. Of the 4310 error responses, 3256 were categorized in a maximum number of nine error strategies for any one task. A total of 372 error responses was regarded as not disorderly, while 617 altogether remained uncategorized.

Although the main purpose of the present study was the investigation of the pattern processing abilities of elementary school children, the recommendations for mathematics educators and classroom teachers put forward in the sixth and final chapter will be influenced by the categorization of the error strategies as set out in Chapter V.

TABLE 41

TEST Q [N:M:C], TEST R [N:M:R] AND TEST [N:M:S]:

CATEGORIZATION OF ERROR STRATEGIES

(N=100)

Error Category	Q [N:M:C]	R [N:M:R]	S [N:M:S]
Response line correct except for			
One inaccuracy	57	40	14
One number associated with an adjacent number	38	30	85
Independent linear sequences			
All four cells	115	110	20
Spoilt by a response associated with an adjacent number	13	0	3
Addition (or subtraction) of 1 to an adjacent line	5	14	47
Each response associated with an adjacent number	64	12	65
Difference between two elements	0	19	1
Total	292	225	235
Not disorderly	48	47	31
Uncategorized	42	117	51
Grand Total	382	389	317

CHAPTER VI

SUMMARY, DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

SUMMARY OF THE INVESTIGATION

The notion of pattern, which has been the focus of this study, is a phenonenom observed in the natural environment and is subsumed in the interpretation the view of mathematics as "the classification and study of all possible patterns (Sawyer, 1955, p.12)." It seems a rational extension of this view that the study of pattern should be a desirable component of the elementary school mathematics curriculum; hence the need for knowledge about the young child's ability in relation to pattern.

The present study was designed primarily to investigate the pattern processing abilities of elementary school children. Five pattern processes (interpolate, continue, reverse, select and generate), two forms (geometric and numerical), and two arrangements (linear and matrix) were incorporated into a hierarchy of 20 tasks, for each of which a test was prepared.

A secondary aim of the study was to determine the relationship, if any, between the children's pattern processing ability and their level of development in Piagetian conservation and classification, two mental operations postulated by the Genevans as necessary for mathematical understanding. Because children tend to move in Piagetian terms from the pre-operational to the concrete-operational stage between the ages of seven and nine years, Grade three classes were selected as the source of the sample.

A further aim of the present study was the investigation of the relationship, if any, between the children's pattern processing ability and their levels of mathematics achievement and intelligence.

Sample

A sample of 100 Grade three children was chosen randomly from 11 classrooms within the Edmonton Public School System. No adaptation classes of specially chosen children were included; however no restriction was placed on children of low intelligence or physical handicap. The sample which consisted fortuitously of 50 girls and 50 boys ranged in age from 92 to 134 months with a mean of 104.5 months. The sample's mean grade score for mathematics achievement was 3.9 while the mean of the intelligence scores was 114, with a range from 73 to 139.

Instrumentation

A battery of tests was administered to each subject either in group situations or individually.

Pattern Processing Test. A test was designed by the investigator for each combination of process, form and arrangement, making a total of 20 tasks. Sixteen tasks, prepared in colour for group administration, n had preset correct answers, whereas the four generate tasks allowed the children to produce their own patterns. The scores obtained were used to assess the children's pattern processing ability (reported in Chapter IV) while the errors were categorized according to strategy and reported in Chapter V.

Piagetian Conservation and Classification. Six tasks derived from research reports published by Piaget and his associates were selected to investigate conservation of number and area, along with multiplicative composition and additive composition of classes. The tasks were presented to each individual child who was then categorized as a conserver, partial conserver or nonconserver, and as a classifier, partial classifier or nonclassifier.

Mathematics Achievement. Mathematics achievement was measured by the two mathematics subtests of the Canadian Test of Basic Skills: Mathematical Concepts and Problem Solving.

Intelligence. Intelligence was measured by the verbal and nonverbal subtests of the Canadian Lorge-Thorndike Intelligence Test, Level A.

Number Knowledge. A simple test devised by the investigator required the children to record the multiples of 2, 3, 4, 5, 6, 8 and 10.

Colour Discrimination. An informal test administered within the Pattern Processing Test asked the children to sort and then label red, green, blue and yellow cardboard shapes.

Results of the Study

A summary of the findings will be presented under the four purposes of the study. A brief survey of the error strategies employed by the children on the Pattern Processing Test concludes the section. From these results are drawn the major findings which will be discussed in the next section of this chapter.

I. Pattern Processing Abilities of Grade Three Pupils

The pattern processing abilities of the Grade three pupils in the sample was judged by their performance on the Pattern Processing Test designed to cover the hierarchy of 20 tasks. The scoring procedure allowed ten points for each task. Among the 16 tasks with preset correct answers, the number of correct responses ranged from 747 for Test D [Geometric: Linear: Select] to 322 for Test P [Numerical: Matrix: Interpolate]. Among the four processes involved in these tasks, the combined scores for the 100 children ranged from 2340 for the select task to 1585 for the reverse task. When accumulated according to form and arrangement, the combined scores decreased gradually from 2222 for the geometric tasks, through 2144 for the geometric matrix and 1845 for the numerical linear tasks, to 1736 for the numerical matrix tasks.

The children's performance on the four generate tasks may be considered from two aspects, the number of scoring items or the points earned by the scoring items. Nearly three-quarters of the possible sequences in both the geometric and linear forms scored points. With a possible total of 200 matrices in each task, the matrix arrangement resulted in 139 scoring for the geometric and 196 scoring for the numerical forms, respectively. The apparent difference between the children's performance on the two matrix tasks may be an artifact of the scoring procedure.

The number of points scored by the generate tasks in the linear arrangement out of a possible 1000 was 421 for the geometric and 451 for the numerical form, respectively. For the matrices generated in the geometric form, 388 points were scored, with 434 points for the numerical form.

Although the results of the Pattern Processing Test outlined above indicate that pattern processing ability exists in this sample of Grade three pupils as a whole, the abilities of the individuals comprising the sample need also to be considered. On every task except the Geometric: Matrix: Reverse, there were children who scored 9 or 10 as well as those who scored 0 or 1, with every intermediate score represented. Every child scored at least five points on some tasks while the greatest number of zero scores gained by any one child was seven.

More than half the possible correlations among the 20 tasks of the Pattern Processing Test were significant (r=.20 to .48). The highest proportion of significant correlations occurred when the tasks were grouped according to process. The correlations obtained were not high and although they support the presence of relationships among the tasks, they do not provide the grounds for confident prediction of performance from one task to another.

When the correlation matrix for the 20 pattern tasks, along with the six Piagetian tasks, mathematics achievement and intelligence subtests, age and number knowledge, was subjected to factor analysis, seven factors appeared. They may be described as

I. Numerical Facility
II. Conservation
III. Geometric Linear Patterning
IV. Pattern Generation
V. Geometric Matrix Patterning
VI. Verbal Intelligence
VII. Part-Whole Relations

The most prominent factor was I. Numerical Facility which included the eight numerical tasks with preset correct answers, the Geometric: Linear: Select task, the mathematics achievement and intelligence subtests and number knowledge. Each of the 20 pattern processing tasks loaded on at least one of the factors I Numerical Facility, III Geometric Linear Patterning, IV Pattern Generation and V Geometric Matrix Patterning. Two factors, II Conservation and VI Verbal Intelligence, were independent of the pattern processing tasks, while the presence of a pattern task in VII Part-Whole Relations was minor.

Hypothesis I was rejected on the grounds that a simple structure of factors was derived from the factor analysis of the 20 tasks of the Pattern Processing Test.

II. Pattern Processing Ability and Conservation

Conservation of number was investigated from two different aspects, linear correspondence and circular correspondence. Of the 100 children in the sample, 82 were categorized as conservers on both tasks with 6 as nonconservers.

Conservation of area situations involved the traditional Barns Task and the Transformed Triangles Task. When the tasks were taken together, there were 61 conservers and 4 nonconservers. The rationalizations expressed by the children in the Barns Task tended to support the contention that this task is perhaps measuring conservation of number for some children. Of the 77 children categorized as conservers, 22 gave clearly numerical rationalizations. Consequently, success on the Transformed Triangles Task was used as the criterion for conservation of area in the statistical analyses.

There was no significant correlation between conservation of number and any pattern processing task for the sample of Grade three pupils. This finding should not be interpreted as evidence that there is no relationship between conservation and pattern processing ability at every stage of a child's development. It may well be that conservation of number is a prerequisite ability for children to process patterns of any description. This question, however, was not investigated in the present study. Hypothesis 2, that there is no significant relationship between Grade three pupils' pattern processing ability and conservation of number, was accepted.

The correlations between conservation of area measured by the Transformed Triangles Task were significant (r=.20 to .32) for 13 of the pattern processing tasks, among which no common relationship seems evident. Moreover, the correlations were low and so are not useful for making predictions about children's pattern processing ability.

Hypothesis 2, therefore, was rejected for 13 of the pattern processing tasks and accepted for the remaining seven (Geometric: Linear: Continue, Reverse and Select; Geometric: Matrix: Interpolate and Continue; Numerical: Linear: Interpolate; Numerical: Matrix: Generate) with regard to conservation of area.

III. Pattern Processing Ability and Classification

Two classification skills were investigated in this study, Additive Composition of Classes by a version of the classic Wooden Beads test and Multiplicative Composition of Classes using circles and squares in two colours.

Of the 100 children in the sample, 44 were categorized as classifiers on the additive composition task, 23 as partial conservers and 33 as nonclassifiers. On the mutiplicative composition task, there were 67 children categorized as classifiers, 30 as partial classifiers and 3 as nonclassifiers. When the two tasks were taken together, 34 children were categorized as classifiers, 65 as partial classifiers and 1 as a nonclassifier.

Between them, the two classification tasks correlated significantly (r=.21 to .37) with 12 different pattern processing tasks, three of the 12 being common to both classification skills. Therefore, to accept Hypothesis 3, that there is no significant relationship between Grade three children's pattern processing ability and classification, seemed justified on the grounds that neither Multiplicative Composition nor Additive Composition correlated significantly with a majority of the pattern processing tasks.

IV. Pattern Processing Ability and Other Variables

The Mathematical Concepts section of the Canadian Test of Basic Skills correlated significantly with every pattern processing task while Problem Solving correlated significantly with 17 of the 20 tasks ($r=.21$ to $.56$), the remaining three being geometric in form (Geometric: Linear: Continue; Geometric: Matrix: Continue and Reverse). Therefore, it seemed justified to reject Hypothesis 4, that there is no significant relationship between Grade three children's performance on the Pattern Processing Test and mathematics achievement.

Verbal intelligence correlated significantly with 17, and nonverbal intelligence with 19 , of the 20 pattern processing tasks ($r=.22$ to $.54$). As all the pattern processing tasks correlated significantly with at least one aspect of intelligence as measured in this study, Hypothesis 4 is rejected in respect to both verbal and nonverbal intelligence.

The significant relationships between the pattern processing ability and mathematics achievement as well as intelligence appear when the sample is treated as a whole. However, a slightly different perspective is gained when individual children's performances are considered in relation to their ratings in mathematics achievement and intelligence. Although there was a tendency for children with high performance on the Pattern Processing Test to have high intelligence and mathematics achievement scores, the reverse was not true. A number of children with high intelligence as well as high mathematics achievement scores could not be regarded as high performers on the Pattern Processing Test. Furthermore, the lowest performers on the Pattern Processing Test had intelligence and mathematics achievement scores distributed over the full range for the sample. Of the 18 children in the sample with intelligence scores below 100, only eight had two or more zero scores on the Pattern Processing Test.

Sex correlated significantly with only one of the pattern processing tasks, Geometric: Linear: Generate, for which the negative correlation (r=-0.23) indicated that being a girl influenced the relationship. It may well be that in their play activities, girls tend to have experience with linear patterns of a spatial kind in the form of decorative craftwork. Age also correlated negatively (r=-0.23) with only one pattern processing task, Geometric: Matrix: Interpolate. In view of these results, Hypothesis 4

9 is accepted for both sex and age.

V. Categorization of Error Responses

A detailed though not exhaustive categorization of error resonses according to strategy was carried out for 12 of the 16 tasks of the Pattern Processing Test with preset correct answers. The geometric matrix tasks were omitted because such a categorization did not seem meaningful. In the remaining 12 tasks the number of possible responses was 11,500. Of the 4310 error resonses, 3256 were categorized in a maximum number of nine error strategies for any one task.

Several error strategies stand out. Altogether 615 error responses in the numerical tasks were classed as being the next number(s) forwards or backwards. This quantity represents 19.1 per cent of the 3225 error responses in the eight numerical tasks. In the numerical linear tasks, the 397 error responses categorized as associative multiples amounted to 21.9 per cent of the 1810 error responses in these four tasks, or 12.3 per cent of all the numerical error responses.

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The error strategy specific to the two reverse tasks in the linear arrangement involved the correct elements in reverse order. The 268 error responses in this category represented 31.0 per cent of the 865 error responses for the two tasks.

In the numerical matrix tasks the strategy type, disloyalty to the given, could be applied either to the information supplied on the task sheet or to the sequence being recorded by the child, however wrong it may be in relation to the given matrix. Altogether 402 (28.4 per cent) of the 1415 numerical matrix error responses demonstrated some form of disloyalty to the given.

The four error strategies labelled as next number, associated multiples, correct elements in reverse order and disloyalty to the given accounted for 1682 or 51.6 per cent of the 3256 error responses categorized according to strategy and 39.0 per cent of the 4310 error responses altogether. A total of 617 error responses remained uncategorized.

DISCUSSION OF THE MAJOR FINDINGS

The major findings of the study are discussed under the following headings:

1. The ability of a sample of Grade three pupils to process pattern;

ii. The significance of two Piagetian tasks in relation to the pattern processing ability of these Grade three children;

iii. The rigidity and logic observed in the error responses to the Pattern Processing Test.

The Ability of Grade Three Pupils to Process Pattern

The ability of Grade three pupils to process pattern can be considered from two angles, firstly, the range of problems involving pattern which the sample as a whole solved with varying degrees of success, and secondly, the characteristics of individuals whose performance on the Pattern Processing Test indicated their identification as successful pattern processors.

The Pattern Processing Test involved five processes, two forms and two arrangements, making a hierarchy of 20 different pattern tasks. The hierarchy of 20 tasks can be partitioned into two groups, the 16 tasks with preset correct answers and the four generate tasks in which the children produced their own patterns.

Within each of the tasks with preset correct answers, examples of different kinds of patterns were presented to the children for solution. The geometric tasks were based on nine different linear pattern descriptions, while the numerical tasks were drawn from 23 different linear pattern types and subtypes. Altogether the children in this study were exposed to a multitude of different patterning problems. The percentages of correct responses for each item of the 16 tasks (see Tables 9, 10, 11 and 12) demonstrate the ability of the sample of Grade three pupils to solve a great variety of specific pattern tasks. At least one-third of the sample responded correctly on three-quarters of the items. There were fewer than five correct responses on only eight of the 120 items. The inappropriate design of the three geometric matrix items (Geometric: Matrix: Reverse #3 and #5; Geometric: Matrix: Select #5) which no child solved has already been acknowledged.

Among the four pattern processes involved in the tasks with preset correct answers the order of difficulty likely to be experienced by the children was conjectured as interpolate, continue, reverse and select, an order not reflected in the number of correct responses. The select task over the two forms and two arrangements proved to be the easiest, the result perhaps of the children's familiarity with workbook exercises and test items of a multiple choice type. With the select task set aside, the interpolate, continue and reverse tasks followed the predicted order of difficulty.

The order of difficulty of the tasks across form and arrangement was proposed more tentatively. The performance of the children, judged by the total number of correct responses for each group of tasks, verifies the predicted order of geometric linear then matrix, followed by numerical linear then matrix. This order of difficulty which appeared when the four blocks of tasks were regarded globally did not necessarily pertain when an individual process was considered. Among the four continue tasks, for instance, the two matrix tasks seemed slightly easier than the two linear tasks. Moreover, the total numbers of correct reponses for each task, arranged from largest to smallest, did not reveal any apparent system. The results of the present study, therefore, do not supply the evidence to confirm that children are likely to experience less difficulty with the simplest numerical items than with the more difficult geometric tasks.

Nevertheless, the performance of the sample on the tasks of the Pattern Processing Test with preset correct answers demonstrates the ability of Grade three pupils to solve a wide assortment of pattern processing problems with varying degrees of success depending on the difficulty component built into the items.

The four generate tasks presented a different kind of problem to the children whose efforts ranged from the production of random lines of elements to sequences and matrices having more complex pattern descriptions than those presented in the tasks with preset correct answers.

Of the total possible 1300 sequences and matrices in the four generate tasks nearly a thousand scored points. In the two geometric tasks, about a quarter of the sequences and matrices displayed randomness partially or completely, with a further 48 examples in the two numerical tasks. Although no child had zero scores in all four generate tasks, two children each had three zero scores each and nine children two zero scores each.

The incidence of randomness in the generate tasks may be due to some children's lack of experience with creative types of activities both in and out of the school. When the children were making their patterns with the coloured shapes, it was observed that some children seemed satisfied to complete the sequence or matrix with no regard for order. The colourfulness of their finished products appeared to be their criterion of success.

With few exceptions, those children who scored consistently well on the four generate tasks were above average scorers on the Pattern Processing Test generally.

However, some children with high scores in some tasks with preset correct answers performed poorly in the generate tasks.

The complex patterns produced by a few individual children give an indication of the potential ability of some Grade three children in pattern processing.

The individual children whose performance on the Pattern Processing Test warrants their identification as successful pattern processors do not easily fit a set of uniform characteristics. However, some trends appear, particularly with regard to variables other than pattern processing. With one exception, the nine children with two or more scores of 9 and 10 on the pattern processing tasks were high achievers on the mathematics and intelligence tests, though the reverse is not true, as noted previously. The group of nine is made up of five boys and four girls so sex seems to play a minor role, if any.

All nine children were beyond the pre-operational stage in both conservation and classification, with half of them operational in all six Piagetian tasks. However, because there were many conservers and classifiers who did not perform well on the Pattern Processing Test as a whole, it must be emphasized once again that being a conserver or classifier on all six Piagetian tasks did not necessarily guarantee success on the Pattern Processing Test.

Although it might be expected that these nine children would score well in the generate tasks, eight of them had scores of 5 or less on at least one of the four tasks. Five of the group earned 9 or 10 points on at least one generate task, with one girl scoring two tens. Success on the generate tasks did not seem to be a critical feature of pattern processing ability.

However, one of the tasks of the Pattern Processing Test appears to be significant. Test P [Numerical: Matrix: Interpolate] had fewer correct responses than any other task. All five children who scored well on Test P performed well on the Pattern Processing Test as a whole. Eight of the nine children regarded as high performers on the Pattern Processing Test had scores of at least 6 on Test P which is nearly twice the mean score of 3.22. The remaining child who earned 2 was well below the rest of the group in intelligence and in mathematics achievement.

Test P [N:M:I] appears to be the only pattern processing task which separated the group regarded as high performers from the rest of the sample. It will be recalled from Chapter V that the error responses to this task seemed to reveal very clearly the error strategies which the children were employing.

A neat check list of the characteristics of a successful pattern processor does not emerge from this study, though high performance on the Numerical: Matrix: Interpolate task, as well as high scores on mathematics achievement and intelligence seem significant.

Significance of Two Piagetian Tasks

The second major finding notes the significance of two Piagetian tasks in relation to the pattern processing ability of Grade three pupils. Two results of the Transformed Triangles Task and Additive Composition of Classes distinguished them from the other four Piagetian tasks for Grade three children. The sample as a whole was almost completely successful on the two conservation of number tasks and Multiplicative Composition of Classes, an indication that Grade three children have possibly reached the ceiling on these tasks. The other three tasks, the Barns Task, the Transformed Triangles Task and Additive Composition of Classes, separated the children into those who could conserve or classify and those who could not do so. The validity of the Barns Task as a measure of conservation of area in all cases is, however, questioned.

On the other hand, the Transformed Triangles Task and Additive Composition partitioned the sample into three groups so that the conservers or classifiers were clearly differentiated from the nonconservers and nonclassifiers, with 7 and 23 children, respectively, being categorized as partially successful. The Transformed Triangles Task correlated significantly (r=.20 to .32) with 13 and Additive Composition with 8 of the pattern processing tasks. Though these quantities do not represent a high proportion, they do provide some statistical information about pattern processing ability at a Grade three level, when many children are passing through the transitional period from the preoperational to the concrete operational stage of development.

It may be recalled that none of the nine children regarded as high performers on the Pattern Processing Test was categorized as preoperational on either of these two tasks. From this study it appears that, unless he has the ability to conserve area along with the class inclusion skill, a child at Grade three level is unlikely to be highly successful in pattern processing.

Rigidity and Logic in Error Strategies

The third major finding of the study arises from the categorization of error strategies employed by the children on the Pattern Processing Test, particularly on the numerical tasks.

Two qualities of the children's mathematical thinking can be inferred from the error strategies four of which predominate: next number, associated multiples, correct elements in reverse order and disloyalty to the given. Rigidity seems to characterize the two most frequent error strategies, next number and associated multiples. The children whose error responses fell into these two categories seemed to be dominated by the process of counting consecutively. This situation may arise because the counting activities experienced by young children are often restricted to the oral recitation of the 1s sequence and workbook exercises for which knowledge of the next consecutive number provides the correct answer.

The use of associative multiples was evidence that the children were no longer restricted to counting by 1s; however, a lack of flexibity still existed in that the children adhered to the idea of counting in groups that would originate at zero. Because of the design of the items, only those children able to recognize numerical relationships of a non-routine though simple nature could be successful on the numerical tasks of the Pattern Processing Test.

On the other hand, the fact that most of the children's error responses could be categorized according to strategy indicates that the children were employing logic of a sort in their problem solving. Bartlett (1958) maintained that his adult subjects did not guess randomly. The same claim may be made for children who employ any of the four predominant strategies as well as those used less frequently such as repetition of part of the sequence given. Altogether 617 of the 4310 responses remained uncategorized. If an exhaustive analysis had been conducted, this number may have been considerably reduced.

The low proportion of error responses remaining uncategorized together with the few error strategies lends support to Collis's (1974) statement that children's answers to mathematical problems are seldom the result of carelessness.

IMPLICATIONS OF SOME OF THE FINDINGS

For Classroom Teachers

Pattern processing provides a rich source of problem solving situations which do not involve reading. The results of this study indicate that children from all normal intelligence levels can be successful, especially as many correct answers are often possible. It was observed during the administration of the Pattern Processing Test that the children seemed to enjoy especially the geometric tasks. When the first of the numerical tasks were presented, a number of children at each school commented on the mathematical nature of the activities, thereby implying that the geometric tasks were not connected with their notion of mathematics. The appeal inherent in colourful geometric pattern activities can be extended to numerical tasks presented attractively and proudly. If the children's reactions of delight to the photographing of their geometric patterns in this study are any guide, the opportunity to create patterns of their own, which are recorded and displayed, may foster more positive attitudes towards mathematics.

In order to reduce the rigidity embodied in the children's error strategies that were demonstrated within this study, teachers may take preventive measures, such as the introduction of group counting starting at a number other than zero, so that more flexible approaches to the solution of problems involving relationships may be developed.

Similarly, teachers may feel encouraged by the limited number of error strategies the children employed in this study to plan a preventive program so that productive rather than unsound problem solving techniques can be developed.

Test P [Numerical: Matrix: Interpolate] proved to be significant within this study for two reasons. Firstly, it was the task which seemed to set apart the children regarded as high performers on the Pattern Processing Test; and secondly, through the categorization of its error responses, the children's strategies were easily identified. Teachers who wish to assess their pupils' progress in pattern processing may find the construction of similar items a useful tool for evaluation purposes. Such a test instrument may also provide the classroom teacher with information about a child's thinking strategies, a guide perhaps to his performance in other areas of mathematics.

Teachers who are concerned about their pupils' knowledge of number facts may look to pattern processing activities as one solution to this pedagogical problem. Rather than resorting to memory drill as the sole means of inculcating number fact mastery, teachers may consider the benefits to be gained from the study and application of pattern. It seems desirable for the child to realize that

> the number system is not simply a haphazard collection of isolated facts, but is highly organized, self-consistent, complex and inter-related. Until the child gains this view of numbers he has not understood the number system. It is the view of the number system as a logical arrangement of numbers that helps to distinguish the mathematician from the drudge who sees numbers as separate and distinct entities. One uses the logic of the system to assist his work; the other struggles with his work because he has not seen the logic (Education Department of Victoria, 1964, #D, p.4).

Teachers may be advised to encourage their pupils not only to look for patterns of the geometrical and numerical kind presented in this study, but also to become aware of the relationships inherent in other facets of elementary school mathematics.

For the Curriculum

Should curriculum developers decide that the study of pattern is a valuable component in elementary school mathematics, this study furnishes them with evidence that Grade three children possess pattern processing ability over a wide range of process, form, arrangement and type. The variation in performance exhibited by the children underlines the necessity that any curriculum change should take into account the differences in ability among children even of the same age and grade level.

The general order of difficulty experienced by this sample of 100 children may suggest a possible sequence of developmental activities designed to expose young pupils to a variety of pattern problems embodying simple relationships of a routine and nonroutine nature. Initially, a small closed system of a limited number of coloured shapes can provide the setting for manipulation of concrete objects so that the children may actively learn not only to solve pattern problems but also to create them. The list of pattern descriptions assembled for both geometric and numerical sequences can serve as a resource, with those items which proved too difficult for the Grade three children in the study acting as pointers towards more difficult relationships that do not require advanced mathematical knowledge.

For Mathematics Educators

Two results of this study apply specifically to mathematics educators who are responsible for the professional preparation of prospective teachers. The young teacher may have studied many aspects of mathematics education yet lack the knowledge of how children think mathematically. In the pattern processing task Numerical: Matrix: Interpolate (Test P), the children revealed through their error strategies some of the quirks in their mathematical thinking. A valuable assignment for prospective teachers might be, for example, the administration of this simple exercise to a group of children, followed by the categorization of the error strategies employed by their particular subjects.

Another difficulty for prospective teachers is the acceptance of individual differences in ability, achievement and maturity found among pupils in an apparently homogeneous class. Although the other four Piagetian tasks may differentiate children at a lower level than Grade three, the Transformed Triangles Task and Additive Composition of Classes separated the children in this study. These simply administered tasks can provide the evidence to convince the prospective teacher that children vary in the ways they think logically, a state of affairs likely to affect their performance in conventional mathematics activities. The knowledge of such differences may influence the young teacher to strive to develop a battery of teaching techniques.

RECOMMENDATIONS FOR FUTURE RESEARCH

This study has concentrated on the products of children's pattern processing. The responses made by the children provided the data not only for the investigation of the pattern processing abilities of Grade three pupils but also for the categorization of error strategies. Donaldson (1963) used a "thinking aloud" technique to explore the problem solving strategies of her subjects. The hierarchy of pattern processing tasks contains blocks of tasks grouped according to process, form and arrangement. It is recommended that the tests prepared for each block of tasks be administered according to the "thinking aloud" technique in order to discover more directly how children operate in mathematical situations involving relationships.

This study focussed on Grade three children. It is recommended that the pattern processing ability of children at other grade levels be investigated, along with the six Piagetian tasks measuring conservation and classification. Such research would reveal whether the same order of difficulty pertains for all elementary school children and also whether the Piagetian tasks other than the Transformed Triangles Task and Additive Composition of Classes relate to pattern processing ability, particularly of younger children.

Another area of research connected with this study is the investigation of other pattern processes, such as copying, transposing and reproducing from memory.

A desirable longitudinal research project would involve the study of pupils for whom pattern is the superordinate topic throughout their elementary school mathematics program.

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APPENDIX A

PATTERN PROCESSING TEST

COLOUR CODE:

B = Blue
G = Green
R = Red
Y = Yellow

SHAPE CODE:

C = Circle
D = Diamond
E = Ellipse (egg)
H = Semicircle (hill)
R = Rectangle
S = Square
T = Triangle
W = Wedge

PATTERN TEST A GEOMETRIC: LINEAR: INTERPOLATE

Materials: Task card in colour
Blank paper for responses

Items:

#1	GC RW GC RW * RW	[GC]
#2	BE YE BS YS * YE	[BE]
#3	RS * BT RS RS BT	[RS or BT]
#4	GC GC YS * GC GC	[YS]
#5	RE GC * RC BE GC	[GC or RE]
#6	RS GS RS BW GS BE * GS RS	[RS]
#7	YC GC YC GE YE GE YC * YC	[GC]
#8	BS BS * RE RE RC BS BS BC	[BC]
#9	GC YS BC * YC BS	[GS]
#10	RT * BT RS YT BS	[YS]

Directions:

Look at the way each row of shapes is made up. Write on your piece of paper the shape with its colour that goes in the space.

Time Allowed: Five minutes

Scoring:
The score is the total number of correct responses.

PATTERN TEST B GEOMETRIC: LINEAR: CONTINUE

Materials: Task card in colour
Blank paper for responses

Items:

# 1	BD YE BD YE BD	[YE,BD]
#2	RD GD RS GS RS GS	[RC,GC]
#3	RS RS YD RS RS YD	[RS,RS]
#4	BE BE RC BE BE	[RC,BE]
#5	GD YT YD GT GD YT	[YD,GT]
#6	GE RE GE YC RE YC GE RE GE	[YC,RE]
#7	YC YC YE GD GD GS YC YC YE	[GD,GD]
#8	RH BH RH BS RS BS RH BH RH	[BS,RS]
#9	YD BH GD YH BD	[GH,YD]
#10	RC BD GC RD BC	[GD,RC]

Directions:

Look at the way each row of shapes is made up. Write on your piece of paper the two shapes with their colours that come next on the right. Place the blank sheet of paper under the right hand edge of the task cards.

Time Allowed: Five minutes

Scoring:

The score is the total number of correct responses each of which consists of two elements in the right order.

PATTERN TEST C GEOMETRIC: LINEAR: REVERSE

Materials: Task card in colour
Blank paper for responses

Items:

#1	[GC,YE] GC YE GC YE GC
#2	[BW,RW] BT RT BW RW BT RT
#3	[YW,GE] YW YW GE YW YW GE
#4	[GC,GC] RS RS GC GC RS RS
#5	[YT,BC] BT YC YT BC BT YC
#6	[BC,GC] YC BC YC GS BC GS YC BC YC
#7	[BS,BC] RW RW RE BS BS BC RW RW RE
#8	[GW,YW] GS YS GS YW GW YW GS YS GS
#9	[GT,BE] RT GE BT RE GT
#10	[GS,RC] YS GC RS YC GS

Directions:

Look at the way each row of shapes is made up. Write on your piece of paper the two shapes with their colours that come next on the left. Place the blank sheet of paper under the left hand edge of the task card.

Time Allowed: Five minutes

Scoring:

The score is the total number of correct responses each of which consists of two elements in the right order.

PATTERN TEST D GEOMETRIC: LINEAR: SELECT

Materials: Task card in colour
Blank paper for responses

Items:

#1	BT RS BT RS BT	A.	RS BT	B.	RS BS
		C.	BT RS	D.	RT BS
#2	GS GS YC YC GS GS	A.	YC YC	B.	GS GS
		C.	YS GC	D.	YS YS
#3	BC RC BS RC RC BS	A.	BC BC	B.	RC RC
		C.	BS RC	D.	RC BS
#4	YT YT GC GC YT YT	A.	YT YT	B.	YT GC
		C.	YC GT	D.	GC GC
#5	BS RW RS BW BS RW	A.	BS RW	B.	BW RS
		C.	RW BS	D.	RS BW
#6	RS BS RS GC RS GC RS BS RS	A.	BS RS	B.	RS BS
		C.	GC RS	D.	RS GC
#7	YE YE YC GS GS YC YE YE YC	A.	YE YE	B.	GS GS
		C.	YC YC	D.	GC YS
#8	BC RC RS BT RT BC RC RS	A.	RC RS	B.	BC RC
		C.	BT RT	D.	BT RS
#9	GE YE GC YS GS YC GE YE GC	A.	GS YC	B.	GE YE
		C.	YE GC	D.	YS GS
#10	BC RT GC BT RC	A.	GC BT	B.	BC RT
		C.	GT BC	D.	BT RC

Directions:

Look at the way each row of shapes is made up. Choose the shape with its colour that you think comes next.

Time Allowed: Five minutes

Scoring:

The score is the total number of correct responses.

PATTERN TEST E GEOMETRIC: LINEAR: GENERATE

Materials: Card marked with four rows of seven collinear cells
Cardboard circles, squares and triangles in red, blue, green and yellow (144 pieces)

Directions:

You have been working with patterns going along in a row. Use your coloured shapes to make patterns of your own.

Recording of Responses: Each child's patterns are photographed in colour.

Time Allowed: Ten minutes

Scoring:

For each sequence, one point for a pattern in the shapes and one point for a pattern in the colours. Two points for complexity beyond the level of alternating shapes and/or colours, or reflection around the middle element.

3

PATTERN TEST F GEOMETRIC: MATRIX: INTERPOLATE

Materials: Task cards in colour
Blank paper for responses

Item #1

BT	YE	BT	YE
YE	[BT]	[YE]	BT
BT	[YE]	[BT]	YE
YE	BT	YE	BT

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the shapes go both ways. Write the shape and its colour that you think goes in each empty space.

Time Allowed: Seven minutes

Scoring:

Within each item, one point if the colour in all four cells is correct, and one point if the shape in all four cells is correct. The score is the total number of points.

#2

BS	BS	YC	YC
YC	[YC]	[BS]	BS
BS	[BS]	[YC]	YC
YC	YC	BS	BS

#3

RT	GE	YT	BE
GE	[RT]	[BE]	YT
RT	[GE]	[YT]	BE
GE	RT	BE	YT

#4

BC	GS	RC	YS
RC	[YS]	[BC]	GS
RS	[YC]	[BS]	GC
BS	GC	RS	YC

#5

RT	YC	RT	YS
BC	[GT]	[BC]	GT
YT	[RC]	[YT]	RC
GS	BT	GC	BT

PATTERN TEST G GEOMETRIC: MATRIX: CONTINUE

Materials: Task cards in colour
Blank paper for responses

Item #1

YE	BT	YE	BT	YE
BT	YE	BT	YE	BT
YE	BT	YE	BT	YE
BT	YE	BT	YE	BT
YE	BT	YE	BT	

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the shapes go both ways. Write the shape with its colour that you think comes next both ways. Place your task card in the top left hand corner of the sheet of paper.

Time Allowed: Seven minutes

Scoring:

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

RH	BH	RD	BD	RH
BD	RD	BH	RH	BD
RH	BH	RD	BD	RH
BD	RD	BH	RH	BD
RH	BH	RD	BD	

#3

GC	YS	GC	YS	GC
RC	BS	RC	BS	RC
GS	YC	GS	YC	GS
RS	BC	RS	BC	RS
GC	YS	GC	YS	

#4

GT	YE	RT	BE	GT
BE	GT	YE	RT	BE
BT	GE	YT	RE	BT
RE	BT	GE	YT	RE
GT	YE	RT	BE	

#5

RT	YC	RT	YS	RT
BC	GT	BC	GT	BC
YT	RS	YT	RC	YT
GS	BT	GC	BT	GS
RT	YC	RT	YS	

PATTERN TEST H GEOMETRIC: MATRIX: REVERSE

Materials: Task cards in colour
Blank paper for responses

Item #1

	RC	GC	RS	GS
RC	GS	RS	GC	RC
RS	GC	RC	GS	RS
GC	RS	GS	RC	GC
GS	RC	GC	RS	GS

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the shapes go both ways. Write the shape with its colour that you think comes next both ways. Place the task card in the bottom right hand corner of the sheet of paper.

Time Allowed: Seven minutes

Scoring:

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

	GC	RC	GS	RS
GC	RS	GS	RC	GC
RS	GC	RC	GS	RS
GC	RS	GS	RC	GC
RS	GC	RC	GS	RS

#3

	RD	BH	BR	GD
GD	YH	YR	RD	YH
BH	BR	GD	BH	BR
YR	RD	YH	YR	GD
GD	BH	BR	RD	BH

#4

	YT	GH	BT	YH
YE	RS	GE	RS	YE
GH	GD	BH	YT	GH
BE	RD	YE	RS	BE
YH	YT	GH	BT	YH

#5

	BD	YT	YD	RT
BS	BC	YS	BC	GS
GT	YD	RT	GD	YT
RS	BC	GS	BC	RS
RT	GD	YT	RD	GT

PATTERN TEST I GEOMETRIC: MATRIX: SELECT

Materials: Task cards in colour
Blank paper for responses

Item #1

YC	GS	YC	GS	GC GS YS YC
YC	GS	YC	GS	YS GC YC GS
GS	YC	GS	YC	GC GS YS YC
GS	YC	GS	YC	GC YC YS GS
YS YC GC GS	YS GC YC GS	YS GC YC GS	GS YC GC YS	

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the shapes go both ways. Choose the shape with its colour that you think comes next both ways.

Time Allowed: Seven minutes

Scoring:
Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

RC	RS	GC	GS	GC GS RS RC
GC	GS	RC	RS	GS RC GC RS
RS	RC	GS	GC	GC RS GS RC
GS	GC	RS	RC	GC RC RS GS
GS GC RC RS	RS GC RC GS	RS GC RC GS	RS RC RC GS	

#3

RC	BS	RC	BS	BC RS BS RC
BC	RS	BC	RS	RS BC RC BS
BS	RC	BS	RC	BC RS BS RC
RS	BC	RS	BC	BC RC BS RS
RS BC RC BS	BS RC BC RS	BS RC BC RS	RS BC RC BS	

#4

YT	RE	BT	GE	GE YE BT YT
GE	BT	RE	YT	GE GT YT RE
BT	GE	YT	RE	YT RE BE BT
RE	YT	GE	BT	RT BT RE GE
YT RE GE RT	GE RT YT RE	GE RE BT YT	GT BT RE GE	

#5

RD	YH	BR	RD	RD YH BR GD
BH	YR	GD	BH	BH RD BH YR
BR	RD	YH	BR	YR YH RD BR
GD	BH	YR	GD	BH YR RD GD
GD YH BH YR	BR GD YH BH	YH YR BR RD	GD YH BR RD	

PATTERN TEST J GEOMETRIC: MATRIX: GENERATE

Materials:

Two cards each marked with a five by five matrix.
Cardboard circles, squares and triangles in red, blue green and yellow (144 pieces)

Directions:

You have been working with patterns going across as well as down. Use your coloured shapes to make patterns of your own.

Recording of Responses: Each child's patterns are photographed in colour.

Time Allowed: Ten Minutes

Scoring:

For each of the two matrices, one point for each linear sequence beyond the first in both the horizontal and vertical directions, for patterns incorporating shape and colour, the three transformations of reflection, translation and rotation being taken into account. Two points for each matrix allocated for complexity in the form of three or more shapes or colours. The score is half the number of points.

PATTERN TEST K NUMERICAL: LINEAR: INTERPOLATE

Materials: Task card in colour
Blank paper for responses

Items:

#1	14 13 12 11 * 9	[10]
#2	3 * 9 12 15 18	[6]
#3	3 8 13 * 23 28	[18]
#4	5 6 * 9 11 12	[8]
#5	36 30 24 * 12 6	[18]
#6	4 * 8 6 12 9	[3]
#7	3 4 6 9 * 18	[13]
#8	23 19 * 11 7 3	[15]
#9	26 20 15 * 8 6	[11]
#10	1 2 * 8 16 32	[4]

Directions:

Look at the way each row of numbers is made up. Write on the piece of paper the number that goes in the space.

Time Allowed: Six minutes

Scoring:

The score is the total number of correct responses.

PATTERN TEST L NUMERICAL: LINEAR: CONTINUE

Materials: Task card in colour
Blank paper for responses

Items:

#1	20 25 30 35 40 45 * *	[50 55]
#2	21 19 17 15 13 11 * *	[9 7]
#3	6 12 18 24 30 36 * *	[42 48]
#4	2 3 5 6 8 9 * *	[11 12]
#5	36 32 28 24 20 16 * *	[12 8]
#6	4 5 8 9 12 13 * *	[16 17]
#7	2 3 5 8 12 17 * *	[23 30]
#8	47 42 37 32 27 22 * *	[17 12]
#9	3 4 6 8 9 12 * *	[12 16]
#10	1 2 4 8 16 32 * *	[64 128]

Directions:

Look at the way each row of numbers is made up. Write on the piece of paper the two numbers that come next on the right. Place the blank sheet of paper under the right hand side of the task card.

Time Allowed: Six minutes

Scoring:

The score is the total number of correct responses each of which consists of two elements in the right order.

PATTERN TEST M NUMERICAL: LINEAR: REVERSE

Materials: Task card in colour
Blank paper for responses

Items:

#1	[4 6]	* * 8 10 12 14 16 18
#2	[20 30]	* * 40 50 60 70 80 90
#3	[5 9]	* * 13 17 21 25 29 33
#4	[4 5]	* * 7 8 10 11 13 14
#5	[30 27]	* * 24 21 18 15 12 9
#6	[90 10]	* * 80 12 70 14 60 16
#7	[33 26]	* * 20 15 11 8 6 5
#8	[47 41]	* * 35 29 23 17 11 5
#9	[24 16]	* * 18 12 12 8 6 4
#10	[4 8]	* * 16 32 64 128 256 512

Directions:

Look at the way each row of numbers is made up. Write on the piece of paper the two numbers that come next on the left. Place the blank sheet of paper under the left hand side of the task card.

Time Allowed: Six minutes

Scoring:

The score is the total number of correct responses each of which consists of two elements in the right order.

PATTERN TEST N NUMERICAL: LINEAR: SELECT

Materials: Task card in colour
Blank paper for responses

Items:

#1	11 22 33 44 55 66	67	77
		65	88
#2	12 18 24 30 36 42	48	43
		46	50
#3	18 19 20 15 16 17	18	16
		12	10
#4	4 8 9 8 14 8	9	8
		18	19
#5	2 5 8 11 14 17	18	20
		21	24
#6	9 5 1 8 5 2	3	10
		5	7
#7	4 6 9 11 14 16	19	18
		17	20
#8	66 57 48 39 30 21	12	20
		22	11
#9	8 10 13 17 22 28	29	40
		32	35

#10 1 2 4 5 10 11 12 22

15 20

Directions:

Look at the way each row of numbers is made up. Choose the number that you think comes next.

Time Allowed: Six minutes

Scoring:

The score is the total number of correct responses.

PATTERN TEST O NUMERICAL: LINEAR: GENERATE

Materials: Blank paper

Directions:

You have been working with patterns going along in a row. Now it's your turn to make five patterns of your own. Use as least six numbers in each pattern.

Time Allowed: Eight minutes

Scoring:

Two points for each pattern according to the following scheme:

0 points:
- a. A repeated sequence
- b. Attempt but
  - i. no pattern obvious
  - ii. insufficient information
- c. Not attempted

1 point:
- a. Sequence of multiples which would originate at 0
- b. Odd numbers
- c. The relationship(s) in one complex sequence repeated in another sequence, for example,

  1, 2, 4, 5, 7, 8

  3, 4, 6, 7, 9, 10

2 points:
A complex sequence, for example,
multiples not originating at 0
Alternating sequences

PATTERN TEST P NUMERICAL: MATRIX: INTERPOLATE

Materials: Prepared response sheet

Item #1

2	4	6
3	[6]	[8]
[6]	[8]	10

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the numbers go both ways. Write the number that you think goes in each empty space.

Time Allowed: Eight minutes

Scoring: For each item, two points if the four elements are correct; one point if two or three elements are correct. The score is the total number of points.

#2

5	10	[15]
7	[12]	[17]
9	[14]	19

#3

10	20	30
[13]	[23]	[33]
16	[26]	36

#4

20	[17]	14
[18]	[15]	12
16	[13]	10

#5

8	[11]	14
13	[16]	[19]
18	[21]	24

PATTERN TEST Q NUMERICAL: MATRIX: CONTINUE

Materials: Prepared response sheet

Item #1

2	4	6	8
3	5	7	9
4	6	8	10
5	7	7	

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the numbers go both ways. Write the number that you think goes in each empty space.

Time Allowed: Eight minutes

Scoring:

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

6	12	18	[24]
10	16	22	[28]
14	20	26	[32]
[18]	[24]	[30]	

#3

23	27	31	[35]
19	23	27	[31]
15	19	23	[27]
[11]	[15]	[19]	

#4

26	23	20	[17]
22	19	16	[13]
18	15	12	[9]
[14]	[11]	[8]	

#5

3	4	5	[6]
6	8	10	[12]
12	16	20	[24]
[24]	[32]	[40]	

PATTERN TEST R NUMERICAL: MATRIX: REVERSE

Materials: Prepared response sheet

Item #1

	3	5	7
4	6	8	10
7	9	11	13
10	12	14	16

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the numbers go both ways. Write the number that you think goes in each empty space.

Time Allowed: Eight minutes

Scoring:

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

	[18]	[20]	[22]
[[illegible]3]	15	17	19
[10]	12	14	16
[7]	9	11	13

#3

	[29]	[31]	[33]
[21]	23	25	27
[15]	17	19	21
[9]	11	13	15

#4

	[9]	[15]	[21]
[7]	11	15	19
[11]	19	23	27
[15]	27	31	35

#5

	[10]	[20]	[40]
[4]	8	16	32
[3]	6	12	24
[2]	4	8	16

PATTERN TEST S NUMERICAL: MATRIX: SELECT

Materials: Prepared response sheet

Item #1

1	2	3	4 6 5 10
3	4	5	7 6 10 9
5	6	7	13 10 8 14
7 10 8 6	7 8 12 10	10 8 9 14	

Directions:

In this kind of puzzle there is a pattern going both ways, sideways and up and down. Try to find how the numbers go both ways. Choose the number you think comes next in each row and column.

Time Allowed: Eight minutes

Scoring:

Each vertical and horizontal line of four correct responses earns one point. The score is the total number of points.

#2

3	6	9	12 15 18 10
5	8	11	20 12 14 19
7	10	13	20 17 14 16
10 8 9 12	18 12 11 20	14 16 20 15	

#3

6	11	16	21 20 17 22
12	17	22	30 27 33 23
18	23	28	29 30 33 35
19 22 20 24	29 24 30 25	30 32 34 39	

#4

13	17	21	25 24 28 22
10	14	18	19 24 20 22
7	11	15	19 17 20 16
5 14 8 4	8 9 22 12	16 13 10 12	

#5

5	10	20	25 21 40 30
7	14	28	29 56 46 30
9	18	36	72 39 37 40
10 16 11 18	22 24 20 19	37 40 46 44	

PATTERN TEST T NUMERICAL: MATRIX: GENERATE

Materials: Two blank four by four grids

Directions:

You have been working with patterns going across as well as down. Now it's your turn to make two patterns of your own.

Time Allowed: Eight minutes

Scoring: Eight points for each matrix according to the following scheme:

1 point: a. Each row and column with a recognizable pattern
b. 16 cells filled by the 1 to 16 sequence
c. One line repeated four times

If each line of one direction is a set of consecutive numbers, two points only are earned, with one point if any one line is not correct. If another sequence appears in the other direction, one further point is recorded.

Points are earned for one direction only if one consecutive sequence is used or if the pattern is identical both ways. If the matrix is correct, one further point is recorded.

Two additional points are allocated for complexity:

1 point: a. A matrix based on a line of 1s but different patterns generated
b. Sequences in reverse order

2 points: a. Different patterns across and down, neither containing a line of 1s
b. Doubling

APPENDIX B

HOLLAND TEST SERIES

HOLLAND TEST SERIES TRANSFORMED TO COLOUR-SHAPE PROBLEMS

(From Klahr & Wallace, 1970, p.244)

Colour: Blue, Green, Red, Yellow;
Shape: Circle, Diamond, Square, Triangle.

#1	BT YS BT YS BT
#2	BD YD BC YC BD YD
#3	BC BC YS BC BC YS
#4	BD YS YD BS YD YS
#5	BD BD YT YT BD BD YT YT
#6	BT YS BT GS YS GD BT YS BT
#7	BS YS BS GC RS GC BS YS BS
#8	BS YS BS YC BC YC BS YS BS
#9	BD BD BC YS YS YC BD BD BC
#10	BD BC BD BS BC BS BD BC BD
#11	BC YC YS BT YT YS BC YC YS
#12	BD YD BC YS BS YC BD YD BC
#13	BS YC GC BS YC GC
#14	BC YC GS BC YC GS
#15	BC YS GT BC YS GT
#16	BC YD GC BD YC
#17	BC GD YC RD GC
#18	BC YD RC BD YC
#19	YS GT BS YT GS
#20	RD GS YD RS GD
#21	GD BC YD GC
#22	RD GS YD RS

#23 RC YT GC RT

#24 RS BD GC RD

APPENDIX C

NUMBER KNOWLEDGE TEST

NUMBER KNOWLEDGE

Count by 2s										
Count by 3s										
Count by 4s										
Count by 5s										
Count by 6s										
Count by 8s										
Count by 10s										